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Continuous interval-valued intuitionistic fuzzy aggregation operators and their applications to group decision making

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ABSTRACT

In this paper, we introduce a new operator called the continuous interval-valued intuitionistic fuzzy ordered weighted averaging (C-IVIFOWA) operator for aggregating the interval-valued intuitionistic fuzzy values. It combines the intuitionistic fuzzy ordered weighted averaging (IFOWA) operator and the continuous ordered weighted averaging (C-OWA) operator by a controlling parameter, which can be employed to diminish fuzziness and improve the accuracy of decision making. We further apply the C-IVIFOWA operator to the aggregation of multiple interval-valued intuitionistic fuzzy values and obtain a wide range of aggregation operators including the weighted C-IVIFOWA (WC-IVIFOWA) operator, the ordered weighted (OWC-IVIFOWA) operator and the combined C-IVIFOWA (CC-IVIFOWA) operator. Some desirable properties of these operators are investigated. And finally, we give a numerical example to illustrate the applications of these operators to group decision making under interval-valued intuitionistic fuzzy environment.

1. Introduction

Multiple attribute group decision making is a usual task in human activities, which consists of finding the most preferred alternative(s) from a given alternative set by a group of experts. In order to choose a desirable solution, decision makers often aggregate their preference information by means of some proper approaches. A very common aggregation method is the ordered weighted averaging (OWA) operator [1], which provides a parameterized family of aggregation operators between the minimum and the maximum. Since coming out, the OWA operator has been employed in a wide range of applications [2–7]. Due to time pressure and lack of data, decision makers may not express their preference information over the alternatives considered precisely [8]. Therefore, the evaluations given by decision makers may be expressed as interval values [9], intuitionistic fuzzy values [10–13] or interval-valued intuitionistic fuzzy values [14–16].

Interval value can be used to describe preference information of decision maker by the left limit and right limit of the interval. In order to aggregate the continuous interval value, Yager [17] introduced a continuous OWA (C-OWA) operator, which is an extension of the OWA operator. Inspired by [17], Yager and Xu [18] developed an extension of the C-OWA operator, called the continuous ordered weighted geometric averaging (C-OWGA) operator. Zhou and Chen [19] extended the C-OWA operator and proposed the continuous generalized OWA (C-GOWA) operator. Other extensions can be found in [20–23]. Intuitionistic fuzzy value (IFV) is an extension of fuzzy number with a membership degree and a nonmembership degree based on the intuitionistic fuzzy set [10]. Up to now, many operators have been proposed for aggregating information [13,24–34]. For example, Xu [13] presented the intuitionistic fuzzy ordered weighted averaging (IFOWA) operator which extends the OWA operator to accommodate the situation where the input arguments are IFVs.

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With the high complexity of socio-economic environments, the group decision making problems are becoming increasingly complex and uncertain while human thinking is fuzzier. As a result, the input arguments are given in the form of interval-valued intuitionistic fuzzy values rather than the intuitionistic fuzzy values in the process of decision making [8,15–16,35–39]. Comparing with IFV, the significant difference is that the membership degree and the nonmembership degree of interval-valued intuitionistic fuzzy value (IVIFV) both are expressed as interval values. However, the IVIFV may be a number with double uncertainty, which will decrease the efficiency of data processing. It is necessary to introduce a new tool to reduce the uncertainty of the IVIFV and improve the accuracy of decision making.

The aim of this paper is to present the continuous interval-valued intuitionistic fuzzy ordered weighted averaging (C-IVIFOWA) operator, which is an extension of the IFOWA operator to the case in which the argument is a continuous IVIFV rather than the IFV. It combines the IFOWA operator with the C-OWA operator by a controlling parameter λ. This combination not only can lower the complexity of data processing, but also can improve the accuracy of decision making.

By proposing the new score function and the accuracy function of IVIFV with respect to λ, we develop a method for the comparison between two IVIFVs. After studying some desirable properties of the C-IVIFOWA operator, we extend the C-IVIFOWA operator to obtain a series of particular operators for aggregation of plenty of IVIFVs, which produce the weighted C-IVIFOWA (WC-IVIFOWA) operator, the ordered weighted C-IVIFOWA (OWC-IVIFOWA) operator and the combined C-IVIFOWA (CC-IVIFOWA) operator. The WC-IVIFOWA operator is an extension of the C-IVIFOWA operator, which combines the C-IVIFOWA operator with the intuitionistic fuzzy weighted averaging (IFWA) operator, and the OWC-IVIFOWA operator is an extension of the WC-IVIFOWA operator which combines the C-IVIFOWA operator with the IFOWA operator. The CC-IVIFOWA operator is an extension of the WC-IVIFOWA operator and the OWC-IVIFOWA operator, which combines the C-IVIFOWA operator with the intuitionistic fuzzy hybrid averaging (IFHA) operator [13]. Moreover, some desirable properties of these operators are investigated including idempotency, boundedness, monotonicity and commutativity.

We also present new approach on the basis of the proposed operators in an example of human resource management decision making. The discussion of different parameter λ on the impact of group decision making result is given. It is possible to consider different values of parameter λ may lead to different decisions. These operators could also be used for other decision making problems such as investment selection, product management, the strategic decision making problem and others.

The remainder is organized as follows. In Section 2, we briefly review some basic concepts such as the intuitionistic fuzzy sets, the interval-valued intuitionistic fuzzy sets, the OWA operator, the C-OWA operator and the IFOWA operator. In Section 3, the C-IVIFOWA operator and its extensions of C-IVIFOWA operator are presented and some of their properties are investigated. New approach is developed in Section 4 and an illustrative example is discussed in Section 5. In Section 6, we come to the main conclusions of the paper.

2. Preliminaries

In this section, we briefly review the intuitionistic fuzzy sets, interval-valued intuitionistic fuzzy sets, the OWA operator and the C-OWA operator.

2.1. Intuitionistic fuzzy sets

Intuitionistic fuzzy sets (IFS) introduced by Atanassov [10] is an extension of the classical fuzzy set, which is suitable to deal with vagueness. It can be defined as follows:

Definition 1. Let a set $X = \{x_1, x_2, \ldots, x_n\}$ be fixed, an IFS A in X is given as follows:

$$A = \{\langle x_i, \mu_A(x_i), v_A(x_i)\rangle | x_i \in X\}, \tag{1}$$

where $\mu_A(x_i)$ and $v_A(x_i)$ represent the membership degree and non-membership degree of the element x_i to the set A for all $x_i \in X$, respectively. The pair $(\mu_A(x_i), v_A(x_i))$ is called an intuitionistic fuzzy value (IFV), and each IFV can be simply denoted as $\alpha_i = (\mu_{\alpha_i}, v_{\alpha_i})$, where $\mu_{\alpha_i} \in [0,1]$, $v_{\alpha_i} \in [0,1]$, $\mu_{\alpha_i} + v_{\alpha_i} \le 1$. Additionally, $S(\alpha_i) = \mu_{\alpha_i} - v_{\alpha_i}$ and $H(\alpha_i) = \mu_{\alpha_i} + v_{\alpha_i}$ are called the score and accuracy degree of α_i, respectively.

For any three IFVs $\alpha = (\mu_\alpha, v_\alpha)$, $\alpha_1 = (\mu_{\alpha_1}, v_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2}, v_{\alpha_2})$, the following operational laws are valid [40]:

(1) $\alpha_1 \oplus \alpha_2 = (\mu_{\alpha_1} + \mu_{\alpha_2} - \mu_{\alpha_1}\mu_{\alpha_2}, v_{\alpha_1}v_{\alpha_2})$;
(2) $\lambda\alpha = (1 - (1 - \mu_\alpha)^\lambda, v_\alpha^\lambda)$, $\lambda > 0$.

To compare any two IFVs $\alpha_1 = (\mu_{\alpha_1}, v_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2}, v_{\alpha_2})$, Xu and Yager [40] introduced a simple method as follows:

(1) If $S(\alpha_1) < S(\alpha_2)$, then $\alpha_1 < \alpha_2$;
(2) If $S(\alpha_1) = S(\alpha_2)$, then
  (a) If $H(\alpha_1) = H(\alpha_2)$, then $\alpha_1 = \alpha_2$;
  (b) If $H(\alpha_1) < H(\alpha_2)$, then $\alpha_1 < \alpha_2$.