Applied Mathematical Modelling 38 (2014) 2235–2248

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Numerical solution of high-order differential equations by using periodized Shannon wavelets

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ARTICLE INFO

Article history:
Received 27 May 2012
Received in revised form 2 September 2013
Accepted 11 October 2013
Available online 30 October 2013

Keywords:
Shannon wavelet
Differential equation
Connection coefficient
Approximation

ABSTRACT

In this paper, periodized Shannon wavelets are applied as basis functions in solution of the high-order ordinary differential equations and eigenvalue problem. The first periodized Shannon wavelets are defined. The second the connection coefficients of periodized Shannon wavelets are related by a simple variable transformation to the Cattani connection coefficients. Finally, collocation method is used for solving the high-order ordinary differential equations and eigenvalue problem. Some equations are solved in order to find out advantage of such choice of the basis functions.

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1. Introduction

In recent years wavelets have been successfully applied to wavelet representation of integro-differential operator.

Shannon wavelets are analytically defined, infinitely differentiable, Cattani connection coefficients can be analytically defined, but they have a slow decay in the variable space and infinite support. In this paper, in order to handle localized problem, periodized Shannon wavelets are used. We solve high-order ordinary differential equations and eigenvalue problem, the efficiency of the method is demonstrated by four numerical examples.

Cattani [1–3] proposed the differential properties of the connection coefficients of Shannon wavelets are explicitly computed with a finite formula up to any order. Shannon–Gabor-wavelet distributed approximate function formula is proposed in [4]. Cattani [5] used Shannon wavelets to defined a method for the solution of integro-differential equations, which method is based on the Galerking method. Cattani and Kudreyko [6] applied periodic harmonic wavelets as basis function in solution of the Fredholm integral equations of the second kind. Maleknejad and Attary [7] used variational iteration technique for the numerical solution of linear and non-linear high order boundary value problems. Siddiqi et al. [8–10] and Siddiqi and Akram [11] used variational iteration technique to solve high-order boundary value problems. Siddiqi et al. [10] used non-polynomial spline to solve 10th-order linear special case boundary value problems. Mai-Duy [12] and Mai-Duy and Tanner [13] used Chebyshev spectral collocation method for directly solving high-order ordinary differential equations. Nielsen [14] used the special properties of wavelets for solving partial differential equations numerically.

The paper is organized as follows. In Section 2, Shannon wavelets and Cattani connection coefficients are given. In Section 3, periodized scaling function and wavelets are given. In Section 4, periodized Shannon wavelets are analytically defined and their corresponding connection coefficients are related to the Cattani connection coefficients. In Section 5, solutions of linear high-order boundary value problems and eigenvalue problem by using periodized Shannon wavelets are discussed. Conclusion is made in Section 6.

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http://dx.doi.org/10.1016/j.apm.2013.10.030

2. Shannon wavelets and Cattani connection coefficients

Following [1], Shannon scaling functions and wavelet functions are defined by

$$\phi(x) = \operatorname{sinc} x = \frac{\sin \pi x}{\pi x},$$

$$\psi(x) = \frac{\sin \pi(x - 1/2) - \sin 2\pi(x - 1/2)}{\pi(x - 1/2)},$$

and for the dilated and translated instances

$$\phi_k^n(x) = 2^{n/2}\phi(2^n x - k) = 2^{n/2}\frac{\sin \pi(2^n - k)}{\pi(2^n x - k)},$$

$$\psi_k^n(x) = 2^{n/2}\frac{\sin \pi(2^n x - k - 1/2) - \sin 2\pi(2^n x - k - 1/2)}{\pi(2^n x - k - 1/2)}.$$

Shannon scaling functions and wavelet functions fulfill the following orthogonality properties [1]

$$\langle \phi_k^0(x), \phi_h^0(x)\rangle = \delta_{kh},$$

$$\langle \phi_k^0(x), \psi_h^m(x)\rangle = 0, \quad m \geqslant 0,$$

$$\langle \psi_k^n(x), \psi_h^m(x)\rangle = \delta_{nm}\delta_{kh}.$$

Next we consider the class of function $f(x)$ such that the following integrals exists and finite

$$\alpha_k = \langle f(x), \phi_k^0(x)\rangle = \int_{-\infty}^{\infty} f(x)\phi_k^0(x)\mathrm{d}x,$$

$$\beta_k = \langle f(x), \psi_k^n(x)\rangle = \int_{-\infty}^{\infty} f(x)\psi_k^n(x)\mathrm{d}x.$$

According to [5], function $f(x)$ and its any order derivative can be reconstructed as follows:

$$f(x) = \sum_{h=-\infty}^{\infty} \alpha_h \phi_h^0(x) + \sum_{n=0}^{\infty}\sum_{k=-\infty}^{\infty} \beta_k^n \psi_k^n(x),$$

and

$$\frac{\mathrm{d}^l}{\mathrm{d}x^l} f(x) = \sum_{h=-\infty}^{\infty} \alpha_h \frac{\mathrm{d}^l}{\mathrm{d}x^l}\phi_h^0(x) + \sum_{n=0}^{\infty}\sum_{k=-\infty}^{\infty} \beta_k^n \frac{\mathrm{d}^l}{\mathrm{d}x^l}\psi_k^n(x).$$

The wavelets decomposition of the any order derivative as follows:

$$\begin{aligned} \frac{\mathrm{d}^l}{\mathrm{d}x^l}\phi_k^0(x) &= \sum_{h=-\infty}^{\infty} \lambda_{kh}^{(l)} \psi_h^0(x), \\ \frac{\mathrm{d}^l}{\mathrm{d}x^l}\psi_k^n(x) &= \sum_{m=0}^{\infty}\sum_{h=-\infty}^{\infty} \gamma_{kh}^{(l)mn} \psi_h^0(x), \end{aligned} \tag{2.1}$$

where

$$\lambda_{kh}^{(l)} = \left\langle \frac{\mathrm{d}^l}{\mathrm{d}x^l}\phi_k^0(x), \phi_h^0(x) \right\rangle, \tag{2.2}$$

$$\gamma_{kh}^{(l)mn} = \left\langle \frac{\mathrm{d}^l}{\mathrm{d}x^l}\psi_k^n(x), \psi_h^m(x) \right\rangle. \tag{2.3}$$

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