Applied Mathematical Modelling 38 (2014) 2290–2295

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Short communication

Remarks on the optimization method of a manufacturing system with stochastic breakdown and rework process in supply chain management

Pin-Shou Ting [c], Kun-Jen Chung [a,b,*]

[a] College of Business, Chung Yuan Christian University, Chung Li, Taiwan, ROC
[b] National Taiwan University of Science and Technology, Taipei, Taiwan, ROC
[c] Department of International Business Management, Shih Chien University, Taipei, Taiwan, ROC

ARTICLE INFO

Article history:
Received 18 June 2012
Received in revised form 15 September 2013
Accepted 8 October 2013
Available online 29 October 2013

Keywords:
Optimization
Manufacturing system
Random breakdown
Rework
Production

ABSTRACT

Chiu et al. (2010) [8] present the proof of convexity of the long-run average cost function $E[TCU(t_1)]$ for a manufacturing system with stochastic breakdown and rework process. This note not only demonstrates that $E[TCU(t_1)]$ is not convex but also adopts the rigorous methods of mathematics to develop the complete solution procedure to find the optimal solution for removing shortcomings of the above paper mentioned.

1. Introduction

It has been argued that the realism of assumptions of EOQ models is challenged by practitioners. However, the EOQ model is still applied industry-wide today [1–4]. The economic quantity (EOQ) model makes the following assumptions (A1) and (A2):

(A1) Items produced are of perfect quantity.
(A2) The manufacturing facilities are reliable.

However, in practice, product quality and manufacturing facilities are not always perfect and reliable. Hayek and Salameh [5] explored the effect of imperfect quality items on the finite production model and allowed the defective items to be reworked at a constant rate. Furthermore, Groenevelt et al. [6] investigated the impact of equipment breakdowns on the operating policy and gained insight into the influences of the occurrence of breakdowns on the lot sizing decision. Chiu [7,8] combined Hayek and Salameh [5] and Groenevelt et al. [6] to study an optimization problem of manufacturing systems with stochastic machine breakdown and rework process. Recently, Chiu [9], Chiu et al. [10], Chiu et al. [11], and Lin and Chiu [12] generalized Chiu [7,8] to discuss the determination of optimal run time for an EPQ model with scrap, rework, and stochastic machine breakdowns.

* Corresponding author at: College of Business, Chung Yuan Christian University, Chung Li, Taiwan, ROC. Tel.: +886 3 2655708; fax: +886 3 2655099.
E-mail address: kunjenchung@gmail.com (K.-J. Chung).

http://dx.doi.org/10.1016/j.apm.2013.10.031

In order to improve the quality of the optimization process of Chiu [7], Chiu et al. [8] presented a proof of convexity of the cost function for such a real-life manufacturing system. Aiming at the above both papers, the main purpose of this note is two-fold:

(P1) This note demonstrates that the cost function discussed in Chiu et al. [8] is not convex. So, it illustrates that the proof of convexity of discussed in Chiu et al. [8] is wrong.

(P2) The fundamentals of mathematics about the solution procedure to locate the optimal solution presented in Chiu [7] and Chiu et al. [8] are not complete.

This note will adopt the rigorous methods of mathematics to develop the complete solution procedure to locate the optimal solution for removing shortcomings of Chiu [7] and Chiu et al. [8].

2. Model formulations

The manufacturing system discussed in this note is the same as those of Chiu [7] and Chiu et al. [8]. The following notation is used in this note.

β	Number of breakdowns per year, a random variable that follows the Poisson distribution,
x	A random defective rate, x is a random variable with known probability density function,
λ	Demand rate (items per unit time),
P	Production rate (items per unit time), ($P > \lambda$),
P_1	Rate of rework of defective items,
K	Setup cost for each production run,
C	Production cost per item (\$/item, inspection cost per item is included),
M	Cost for repairing and restoring the machine,
C_R	Repair cost for each defective item reworked (\$/item),
h	Holding cost per item per unit time (\$/item/unit time),
h_1	Holding cost for each reworked item per unit time (\$/item/unit time), ($h_1 \geq h$),
t	Production time before a random breakdown occurs,
t_1	The optimal production run time (i.e. production uptime) to be determined,
T	Cycle length whether a machine breaks down or not,
$TCU(t_1)$	The total production–inventory costs per unit time whether a breakdown takes place or not,
$E[TCU(t_1)]$	The expected total inventory costs per unit time whether a breakdown takes place or not,
t_1^*	The optimal solution of $E[TCU(t_1)]$,
$\in$	Belongs to.

Referring to equation (23) in Chiu [7] or equation (1) in Chiu et al. [8], we find that the expected total cost function $E[TCU(t_1)]$ can be expressed as follows:

$$E[TCU(t_1)] = \frac{K\lambda\beta}{P(1-e^{-\beta t_1})} + \frac{M\lambda\beta}{P} + C\lambda + C_R E[x]\lambda + \omega\left[\frac{-t_1 e^{-\beta t_1} - \frac{e^{-\beta t_1}}{\beta} + \frac{1}{\beta}}{1-e^{-\beta t_1}}\right], \tag{1}$$

where $\omega = hP - h\lambda + \frac{P\lambda E[x^2]}{P_1}[h_1 - h] > 0$. Eq. (1) yields

$$\frac{dE[TCU(t_1)]}{dt_1} = \frac{e^{-\beta t_1} a(t_1)}{(1-e^{-\beta t_1})^2}, \tag{2}$$

$$\frac{d^2 E[TCU(t_1)]}{dt_1^2} = \frac{\beta e^{-\beta t_1} m(t_1)}{(1-e^{-\beta t_1})^3}, \tag{3}$$

where,

$$a(t_1) = \frac{-K\lambda\beta^2}{P} + \omega[\beta t_1 + e^{-\beta t_1} - 1],$$

$$m(t_1) = \frac{K\lambda\beta^2}{P}(1+e^{-\beta t_1}) + 2\omega(1-e^{-\beta t_1}) - \beta\omega t_1(1+e^{-\beta t_1}).$$