Applied Mathematical Modelling 38 (2014) 2296–2301

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Short communication

On the convexity for the expected total cost per unit time of the EPQ model with scrap, rework and stochastic machine breakdown

Pin-Shou Ting [c], Kun-Jen Chung [a,b,*]

[a] College of Business, Chung Yuan Christian University, Chung Li, Taiwan, ROC
[b] National Taiwan University of Science and Technology, Taipei, Taiwan, ROC
[c] Department of International Business Management, Shih Chien University, Taipei, Taiwan, ROC

ARTICLE INFO

Article history:
Received 12 September 2012
Received in revised form 25 July 2013
Accepted 8 October 2013
Available online 30 October 2013

Keywords:
EPQ
Optimization
Convexity
Production
Breakdowns
Rework

ABSTRACT

This study explores the economic production quantity model with scrap, rework and stochastic machine breakdown. The main purpose of this paper is twofold:

(P1) This paper will adopt the rigorous methods of mathematics to demonstrate that the expected total cost per unit time is convex on all positive numbers to improve the conditional convexity in Theorem 1 of Chiu et al. (2010) [7].
(P2) This paper gives the concrete proof to provide bounds for the optimal production run time to remove the logical shortcomings of mathematics presented in proof of Theorem 2 of Chiu et al. (2010) [7].

1. Introduction

In real life manufacturing systems, generation of defective items and random breakdown of production equipment are inevitable. Hence, many researchers are concerned with determination of optimal run time for an economic production quantity (EPQ) model with scrap, rework, and stochastic machine breakdowns. Related articles can be found in [1–13] and their references.

Recently, Chiu et al. [4] employs mathematical modeling along with a recursive searching algorithm to determine the optimal run time for an imperfect production rate model with scrap, rework, and stochastic machine breakdown. Basically, Chiu et al's model is interesting. However, this paper indicates that their search algorithm has shortcomings. So, the main purpose of this paper is twofold:

First, this paper will adopt the rigorous methods of mathematics to demonstrate that

the expected total cost per unit time is convex on all positive numbers to improve the conditional convexity in Theorem 1 of Chiu et al. [4]. Secondly, it gives the concrete proof to provide bounds for the optimal production uptime to remove the logical shortcomings of mathematics presented in proof of Theorem 2 of Chiu et al. [4].

* Corresponding author at: College of Business, Chung Yuan Christian University, Chung Li, Taiwan, ROC. Tel.: +886 3 2655708; fax: +886 3 2655099.
E-mail address: kunjenchung@gmail.com (K.-J. Chung).

0307-904X/$ - see front matter
http://dx.doi.org/10.1016/j.apm.2013.10.047

2. Formulation of the Model

The manufacturing system discussed in this paper is the same as that of Chiu et al. [4]. The following notation will be used in the whole paper.

β	Expected number of breakdowns per year, the mean of a random variable that follows the Poisson distribution,
x	A random defective rate, x is a random variable with known probability density function with $0 \le E[x] \le 1/3$,
θ	Portion of defective items to be scrapped before the reworking,
$1-\theta$	Portion of defective items to be reworked and repaired,
λ	Demand rate (items per unit time),
P	Production rate (items per unit time), ($P > \lambda$),
P_1	Rate of rework of defective items,
K	Setup cost for each production run,
C	Production cost per item ($/item, inspection cost per item is included),
M	Cost for repairing and restoring the machine,
C_R	Repair cost for each defective item reworked ($/item),
C_S	Disposal cost per scrap item ($/item),
h	Holding cost per item per unit time ($/item/unit time),
h_1	Holding cost for each reworked item per unit time ($/item/unit time), ($h_1 \ge h$),
t_1	The production run time (i.e. production uptime) to be determined,
T	Cycle length whether a machine breaks down or not,
$TCU(t_1)$	The total production-inventory costs per unit time whether a breakdown takes place or not,
$E[TCU(t_1)]$	The expected total cost per unit time whether a breakdown takes place or not,
t_1^*	The optimal run time of $E[TCU(t_1)]$,
$\in$	Belongs to,
$\notin$	Does not belong to.

Following the above notations and assumptions, Chiu et al. [4] reveal that the expected total cost per unit time $E[TCU(t_1)]$ can be expressed as follows:

$$E[TCU(t_1)] = \left[\frac{\lambda\{C + C_R E[x](1-\theta) + C_S E[x]\theta - hg\}}{1-\theta E[x]}\right] + \frac{\delta t_1}{2[1-\theta E[x]]} + \frac{\lambda}{[1-\theta E[x]]}\left\{\frac{K}{Pt_1} + hg\theta E[x](1-e^{-\beta t_1}) + \left[\frac{M}{P} + \frac{hg}{\beta}\right]\frac{(1-e^{-\beta t_1})}{t_1}\right\} \tag{1}$$

where $\delta = hP(1 - 2\theta E[x] + \theta^2 E[x^2]) - h\lambda + 2h\lambda\theta E[x] + \frac{P\lambda(1-\theta)^2 E[x^2][h_1 - h]}{P_1}$.

$$= hP(1+\theta^2 E[x]) + h(P-\lambda)(1-2\theta E[x]) + \frac{P\lambda(1-\theta)^2 E[x^2][h_1-h]}{P_1} > 0,$$

since $0 \leqslant E[x] \leqslant 1/3$ and $0 \leqslant \theta \leqslant 1$, we have $1 - 2\theta E[x] > 0$. Eq. (1) yields the first-order and second-order derivatives of $E[TCU(t_1)]$ with respect to t_1 as follows:

$$\frac{dE[TCU(t_1)]}{dt_1} = \frac{a(t_1)}{[1-\theta E[x]]t_1^2}, \tag{2}$$

$$\frac{d^2E[TCU(t_1)]}{dt_1^2} = \frac{\lambda m(t_1)}{[1-\theta E[x]]t_1^3}, \tag{3}$$

where

$$a(t_1) = \frac{\delta t_1^2}{2} + \lambda\left\{\frac{-K}{P} + hg\theta E[x]\beta t_1^2 e^{-\beta t_1} + \left[\frac{M}{P} + \frac{hg}{\beta}\right]\left[e^{-\beta t_1} - 1 + \beta t_1 e^{-\beta t_1}\right]\right\}, \text{and} \tag{4}$$

$$m(t_1) = \frac{2K}{P} - hg\theta E[x]\beta^2 t_1^3 e^{-\beta t_1} + \left[\frac{M}{P} + \frac{hg}{\beta}\right]\left[2(1-e^{-\beta t_1}) - 2\beta t_1 e^{-\beta t_1} - \beta^2 t_1^2 e^{-\beta t_1}\right]. \tag{5}$$