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# A new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates

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ABSTRACT

A new hyperbolic shear deformation theory applicable to bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates is presented. This new theory has five degrees of freedom, provides parabolic transverse shear strains across the thickness direction and hence, it does not need shear correction factor. Moreover, zero-traction boundary conditions on the top and bottom surfaces of the plate are satisfied rigorously. The energy functional of the system is obtained using Hamilton's principle. Analytical solutions of deflection and stresses are obtained using Navier-type procedure. Free vibration frequencies are then accurately calculated using a set of boundary characteristic orthogonal polynomials associated with Ritz method. Numerical comparisons are conducted to verify and to demonstrate the accuracy and efficiency of the present theory. Excellent agreement with the known results in the literature has been obtained.


## 1. Introduction

Functionally graded materials (FGMs) are a new kind of inhomogeneous composite materials which have a smooth and continuous variation of material properties along one or more directions. At each interface, the material is chosen according to specific applications and environment loadings. Gradually varying the material properties can prevent from interface cracking, disbonding and residual stresses and thus maintain structural integrity to a desirable level. These highly heterogeneous materials are also used as thermal barrier, wear coating and corrosion resistant coating in many engineering applications.

In 1984–1985, a group of Japanese scientist proposed, for the first time, the concept of FGM. Five years later, the first international conference [1] was held at Sendai-City in Japan. The interest was such that a rapid progress of FGM research in Japan was noticed from 1984 to 1996 [2]. Since then, the increased attention given by many industrial engineers and researchers to these new composite materials was motivated by the real possibilities of improvement that can be achieved for the mechanical properties of composites to overcome problems arising in composite structures such as delamination due to high thermal gradients. Nowadays, these innovative materials are increasingly being used in the manufacture of aerospace and aeronautical components designed to support high temperature environments. They are also used in the manufacture of

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components that serve critical functions in aircrafts, space rockets, launchers and space shuttles. Further extensive applications of FGMs can be found in nuclear reactors, chemical plants, and in other applications such as biomechanical, optical, automotive, electronic, mechanical, civil, and shipbuilding industries.

FGMs are typically made of a ceramic because of its thermal stability, good thermal shock resistance, mechanical strength, high-temperature creep resistivity, low density, excellent chemical resistance, good stiffness-to-weight ratio, and a metal because of its good fracture toughness. The functional association of these two high performance materials provides to aviation and aerospace manufacturers the ability to face successfully a growing demand in achieving optimal performance while satisfying stringent safety standards. A good overview of the concept of FGMs, their processing, and their applications can be found in Refs. [3–5].

In order to solve problems of structures composed of structural elements it is necessary to find the right theory that describes correctly the static and dynamic behaviors of the structure. This has led many researchers to develop more theories as well as new and effective methods of resolution.

On basis of Euler–Bernoulli beam theory that relies on plane cross-sections hypothesis Kirchhoff [6] built a new theory for thin plates in 1850, on the assumption that normals to the undeformed midplane remain straight and normal to the deformed midplane and unstretched in length. It was then extended to thin shells by Love [7] in 1888. These two theories together are referred as classical theory of plates and shells (also called Kirchhoff–Love theory). Since this theory neglects the transverse shear flexibility, this leads to an overestimation of the bending stiffness and consequently an overestimation of vibrational frequencies and underestimation of deflections.

The theory of moderately thick plates, known as first order shear deformation plate theory (FSDPT), was consolidated by Mindlin from the works of Rayleigh [8], Timoshenko [9,10], Reissner [11] and Uflyand [12]. The works of Mindlin [13] were based, mainly, on observations made before by Rayleigh and Timoshenko. In fact, Rayleigh stated in 1877, that it is important to include the rotary inertia in any vibration analysis system. The works of Timoshenko [9,10] have shown that taking into account the effects of rotary inertia and transverse shear affects the natural frequencies of bending beams. These two effects tend to reduce the calculated resonant frequencies, due to the increase in inertia and flexibility of the system. Since the first order theory of plates do not satisfy the boundary conditions at the top and bottom surfaces of the plate (zero traction boundary conditions), a shear correction factor across the thickness is necessary for FSDPTs [9,10,13]. The value of the shear correction factor depends on the geometry of the plate, on the variation of Poisson's ratio through the thickness, on the applied loading and on boundary conditions. Around 1970, Whitney [14], Whitney and Sun [15], Pagano [16] and Srinivas and Rao [17] have shown, through their respective works, that the effects of transverse shear are more important for laminated plates than for isotropic plates. In the literature, three approaches of approximation models for moderately thick plates were reported [13,18–21]. Reissner's approach [22] involves a stress–displacement field containing in-plane normal and shear stresses. The approach of Henchy–Mindlin [8] assumes a displacement field, while the approach of Ambartsumyan [21] assumes a field of normal and shear stresses.

It is well known that the effect of transverse shear deformation and normal stress in the thickness direction becomes important above a certain value of the thickness-to-side ratio. To avoid the use of a shear correction factor and to rely on more realistic assumptions than those of Kirchhoff and Mindlin theories, new mathematical models were needed. The aim of these models is to better capture static and dynamic behaviors of plates. To address this challenging need, several refined theories have been proposed by various authors [23–34], which are all based on the expansion of displacements through the thickness using power series (see [35,36]). Most of the existing higher order shear deformation theories are based on approaches of Reissner, Henchy–Mindlin and Ambartsumyan and are applicable to various plates' models (e.g. [20,23–28,30,31,37–41]), while some are applicable to multilayer FGM plates as well as to thick FGM plates (e.g. [15,28,42–50]). The unified formulation (known as CUF) proposed by Carrera and Petrolo [51,52] was recently extended by the noteworthy works of Demasi [53–57]. CUF permits a systematic assessment of a large number of plate and shell models, whose accuracy has been well demonstrated in the literature.

New formulations involving higher order shear deformation theories have emerged in recent years for static and dynamic analysis of beams and plates. Among all these new theories, Aydogdu [58] proposed a new exponential shear deformation theory for laminated composite plates. El Meiche et al. [59] developed a new hyperbolic shear deformation theory for buckling and vibration of FG sandwich plates. Most of new emerging higher order theories rely on five unknowns leading to five governing equations [28,29,33,34,49,58–64]. They all neglect the thickness stretching by considering the transverse displacement independent of the thickness coordinate (transverse inextensibility), contrary to new quasi-3D hyperbolic and sinusoidal shear deformation theories developed recently by Neves et al. [65,66]. In their works, the out-of-plane displacement is defined as quadratic in the thickness direction while the in-plane displacements are of hyperbolic sine or sinusoidal type, leading to best predicting static and dynamic behaviors of plates at the cost of solving a system of nine governing equations. Ghugal and Sayyad [67] studied the free vibration of thick orthotropic plates involving trigonometric shear deformation theory. In-plane displacements are of sine type while the transverse displacement is of cosine type. According to these authors, their theory is not only capable to produce frequencies of thickness stretch mode, but also yields the exact value of dynamic shear correction factor from the thickness shear motion of vibration. It is interesting to note that Kant [47], when studying bending behavior of isotropic plate, adopted a formulation that incorporates a quadratic variation of transverse shearing strains and a linear variation of transverse normal strain through the thickness of the plate besides the use of the three-dimensional Hooke's law.