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Nonlinear dynamics modeling and analysis of two rods connected by a joint with clearance

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ABSTRACT

In this paper, the nonlinear dynamic behaviors of two elastic rods connected by a joint with clearance are investigated. The rods and joint system are modeled by two degrees of freedom of mass-spring system with a clearance. The equations of motion of the mass-spring system with clearance are established by means of d'Alembert's principle. Due to the nonlinearity caused by clearance, the dynamic properties of the system are studied using the averaging method and compared with numerical solutions. The frequency responses of the system subjected to cosinusoidal excitation are obtained as well as the effects on the vibration characteristics induced by different gap sizes are investigated. The stability condition for steady solutions is presented based on Lyapunov theory. The method for detecting the multi-value performances of the frequency response has been proposed. Based on this method, the effects of the clearance size on the multi-value response characteristics are investigated, and the critical value of the clearance is obtained.

1. Introduction

Mechanisms including joints with clearance are extensively applied in spatial structures especially in deployable trusses such as solar panels, satellite antennas and space stations. Some clearances are necessary due to the components assemblage or to allow the relative motion between the connecting bodies as well. While the manufacturing tolerances, wear and material deformations can also cause the clearance between the journal and the bearing. Clearances can be a source of impact forces resulting machining wear and tolerance between the components in the structure. In these cases, the impulsive force caused by the excessive joint clearances can lead to the degradation of the performance of the mechanical systems. Hence the effects of joint clearance are capturing the attention of a large number of researchers [1–3].

Many of these works are focus on how to model the joint with clearance more precisely. Generally, since the concept of the impact pair proposed by Dubowsky and Freudenstein [4,5], there are three main modeling methods for joint with clearance, i.e. the massless link approach [6], the spring-damping approach [7,8] and the momentum exchange approach [9]. These modeling strategies are applied extensively for different working conditions to simulate the motion of the system as closely as possible.

From the view of the nonlinear dynamics, the vibration of the structure with clearance shows typical nonlinear behaviors. And the system is a non-smooth one which might be called as Filippov system [10] with plenty of nonlinear dynamical

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properties [11]. The number and the stability of the periodic solutions may change with the variation of the clearance, which can lead to various types of bifurcation such as pitchfork bifurcation [12], Hopf bifurcation [13] as well as discontinuous bifurcation [11,14]. Many methods are developed to solve this problem analytically and numerically [15–17], such as the harmonic balance method, Runge–Kutta algorithm and shooting method. Numerical methods may obtain the time domain response more conveniently and can solve systems with much more degrees of freedom than the approximately analytical methods, while the detailed parametric analysis for dynamic behaviors is often neglected. Thus, several analytical methods are developed to acquire the insights into the influences of structural parameters on the nonlinear dynamic characteristics. And the averaging method used in the present work is just an analytical method which can study these problems in phase space.

In this paper, two rods connected by a joint with clearance subjected to axial harmonic excitation are taken into account. The equations of motion of the rods and joint system for its axial vibration are established. Due to the nonlinearity caused by the clearance, the averaging method is applied to obtain the steady solutions directly instead of introducing smoothening functions which are frequently used in the harmonic balance technique. The influences of the parameters in the structure especially the size of clearance on the nonlinear dynamic behaviors of the structural system are investigated. The stability condition for steady motions is constructed based on Lyapunov theory.

2. Problem formulation for analytical study

Connecting rods system including joints with clearance can be seen extensively in the engineering mechanisms especially in the truss structures. Fig. 1(a) shows the schematic diagram of two rods connected by a joint with clearance subjected to axial excitation. This structure is an infinite degrees of freedom mechanical system. In order to investigate the essential nonlinear dynamic properties caused by the clearance and obtain the analytical solutions for the vibration of the structure by using of the averaging method, simplifications are obviously required. Lumped mass method has been used to simplify many types of continuous structures such as rods, beams and pipes. It is necessary to point out that this method has also been used to analyze the nonlinear vibration performances by many researchers [18,19] rather than the linear problems. Based on these facts, the dynamic model for the rods and joint system with clearance is modeled by the lumped mass method in this analysis and shown in Fig. 1(b).

In the simplified model, m_1 and m_2 denote the concentrated masses of the rods and joint, respectively. k_1 is the equivalent stiffness of the rod 1. Unlike the linear model, there is one non-linear stiffness force in this model, i.e. $k_2 f(x_1,x_2)$, in which k_2 is a constant representing the equivalent stiffness of the rod 2 and $f(x_1,x_2)$ is referred as a clearance type non-linearity as shown in Fig. 2 where δ denotes the size of clearance of the joint, and it is obvious that $\pm\delta$ are stiffness break points.

The vibration region which can be divided into three subregions is changed instantly due to the switch of the structural parameters. When $-\delta < x_2 - x_1 < \delta$ (case 1), it means that there is no contact between the two mass elements. For the case of $x_2 - x_1 < -\delta$ (case 2), it can be viewed as the left mass element passing through the clearance from left to right and contacting the right spring during the vibration. And the state of motion for $x_2 - x_1 > \delta$ is similar to the case 2, while the difference is that the left mass element passes through the gap in the opposite direction.

The parameters in mass-spring system are equivalent to the rods and joint system according to the principles in Mechanics of Materials. Since it is difficult to determine the structural damping coefficients of rods, an important and extensively applied approximation is adopted to obtain the equivalent linear viscous damping [20,21]. The equivalent principle is to deem that the energy dissipation of these two types of damping is equal to each other over one period of vibration. For rod 1, the work done by equivalent linear viscous damping is $W_d = -\pi\omega c_1 A^2$. Since the damping mainly takes effect in the resonance region. It is reasonable to take $\omega = \omega_{1n}$, where ω_{1n} is the natural frequency of rod 1. Hence,

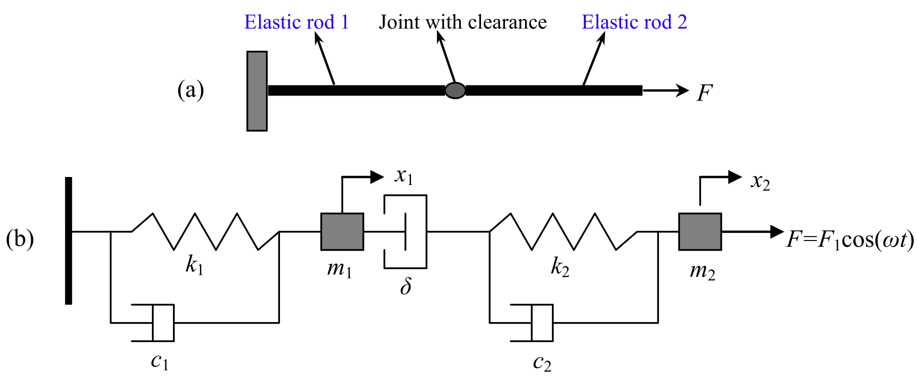

Fig. 1. Schematic diagram of two rods connected by a joint with clearance (a), and the two degrees of freedom mass-spring model for the rods and joint system with clearance (b).