Applied Mathematical Modelling 39 (2015) 2633–2648

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Modeling the instability of electrostatic nano-bridges and nano-cantilevers using modified strain gradient theory

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ARTICLE INFO

Article history:
Received 24 July 2013
Received in revised form 2 July 2014
Accepted 12 November 2014
Available online 22 November 2014

Keywords:
Pull-in instability
Van der Waals force
Size effect
Modified strain gradient theory
Nano-cantilever
Nano-bridge

ABSTRACT

Experimental observations reveal that the physical response of nano-structures is size-dependent. Herein, modified strain gradient theory in conjunction with Euler–Bernoulli beam theory have been used for mathematical modeling of the size dependent instability of nanostructures. Effect of van der Waals intermolecular force has been included in the mathematical model. Two common beam-type systems including double-clamped nano-bridge and clamped-free nano-cantilever have been investigated. Three approaches including using an approximated differential transformation method (DTM), applying iterative numerical method and developing a simple lumped parameter model have been employed to solve the governing equations of the systems. The obtained results have been compared with those obtained via numerical method. Furthermore a lumped parameter model has been developed to simply explain the physical performance of the systems without mathematical complexity. The pull-in parameters of the nanostructures as basic design parameters have been calculated. Effect of the size dependency on the pull-in performance has been discussed for both nano-structures.

1. Introduction

Emerging revolution of nanotechnology gives the opportunity to develop high performance precise ultra-small systems for engineering applications. Recently, micro/nano-electromechanical systems (MEMS/NEMS) has found enormous applications in many science branches e.g. engineering, chemistry, optic, magnetic, electronics, etc. A beam-type MEMS/NEMS constructed from two conductive electrodes, which one electrode is movable and the other one is fix (ground). Applying voltage between the electrodes leads to deformation of the movable electrode toward fixed electrode. When electrostatic force exceeds the elastic resistance of the beam, the instability occurs and movable electrode suddenly adheres to the ground. Instability characteristics of MEMS in micro-scales has been investigated by previous researchers during previous decade [1,2]. With decreasing the dimensions to sub-micron, the nano-scale phenomena appear that should be considered in theoretical models.

One of the most important nano-scale issues is the presence of van der Waals (vdW) force that affects the stability of nano-structures in nano-scale distances. This attraction can significantly influence the NEMS performance when the initial gap between the components of nanostructure is typically below several ten nanometers. In this case, the attraction between

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http://dx.doi.org/10.1016/j.apm.2014.11.011

two surfaces is proportional to the inverse cube of the separation and is affected by material properties [3–6]. In recent years, various approaches such as finite difference methods [3,4], finite element methods [5,6], developing simple lumped parameter models [7,8] are utilized to investigate the effect of vdW force on performance of nano-systems.

Another important phenomenon in nano-scale is size dependency of the mechanical performance of nano-structures. It is experimentally well established that elastic characteristics of materials highly are affected by the dimensions of the structure. It has been shown that torsional hardening of copper wire increases by a factor of 3 as the wire diameter decreases from 170 to 12 μm [9]. Stolken and Evans [10] showed that decrease in thickness of thin nickel beams from 50 to 12.5 μm can lead to great increase in the plastic work hardening of the constitutive material. Also, the size-dependent behavior has been detected in some kinds of polymers [11]. For hardness measurement of bulk gold, it is found that the plastic length scale parameter (for indentation test and hardness behavior) of Au increases from 470 nm to 1.05 μm with increasing the Au film thickness from 500 nm to 2 μm [12]. Using micro bend testing method, the plastic intrinsic material length scale of 4 μm for copper and 5 μm for nickel were determined [13]. All these experiments imply that when the characteristic size (thickness, diameter, etc.) of a micro/nano element is in the order of its intrinsic the material length scales (typically sub-micron), the material elastic constants highly depend on the element dimensions. The size-dependent behavior of materials and structures at sub-micron distances cannot be modeled using classical continuum mechanics. However, by applying non-classic continuum theories, the size dependent behavior of nano-structures is attributed to material length scale parameters. A length scale parameter might be considered as a mathematical parameter that scales the strain gradients in the constitutive model so as to balance the dimensions of strains (ε) and strain gradients ($d\varepsilon/dx$) [13]. As the characteristic length of the deformation field becomes significantly larger than the material length scale parameter, strain gradient effects become negligible because the strain terms are much larger than their scaled gradient terms [13]. In this regards, the non-classical theories such as non-local elasticity [14], couple stress theory [15], strain gradient theory [16], modified strain gradient theory [17], modified couple stress theory [18], etc have been developed to consider the size effect in theoretical continuum models. One of the pioneering works in modeling the size-dependent behavior of micro-structures was conducted by Cosserat, in the beginning of 20th century [19]. Afterwards more general continuum theories have been developed for linear elastic materials in which gradients of normal strains were included and additional material length scale parameters were therefore added as well as Lame constants [20–22]. The first strain gradient theory was introduced by Mindlin and Eshel [23] in which the potential energy–density assumed to be depended on the gradient of stain as well as strain. The most comprehensive work was done by Mindlin [22] which contained five additional material parameters and encompassed other non-local theories as special cases. In the strain gradient theory [16], the second-order deformation gradient tensor is decomposed into two independent parts, the stretch gradient tensor and the rotation gradient tensor. Lam et al. [17], introduced a modified strain gradient theory with three material length scale parameters relevant to dilatation gradient, deviatoric gradient, and symmetric rotation gradient tensors. In recent years, modified strain gradient theory used by many researchers to analysis of static and dynamic behavior of micro/nano beams. A simple form of the modified strain gradient theory with only one size parameter i.e. modified couple stress theory has been used by previous researchers to analyze the pull-in behavior of NEMS [24–31]. However, little attention has been paid for modeling the pull-in instability of the micro/nano devices using general form of modified strain gradient theory. Wang et al. [32] presented a size-dependent model for electrostatically actuated micro-beams using strain gradient elasticity theory ignoring the effect of intermolecular forces. They used generalized differential quadrature method to numerically solve the governing equation. In other works [33,34] the pull-in instability of rectangular and circular plate MEMS have been studied using strain gradient theory. Ansari et al. [35] and Mohammadi et al. [36] studied dynamic pull-in instability and free vibration characteristics of circular microplates subjected to the combined hydrostatic and electrostatic forces are investigated.

In this work, effect of vdW force on the size-dependent pull-in instability of nano-bridges and nano-cantilevers is investigated. It should be noted that none of the above mentioned works has taken the important effect of nano-scale forces into account. Indeed, to the best knowledge of the authors, there are no works have been used modified strain gradient theory for modeling the interaction between vdW force and the size-effect in beam-type NEMS. In Refs. [31,37] the pull-in behavior of NEMS was studied using modified couple stress theory neglecting the effect of mid-plane stretching. However, in this paper a more generalized theory (modified strain gradient) is employed which introduce three length scale parameters and the stretching is considered in the theoretical model. Furthermore, three different approaches including using differential transformation method, developing a lumped parameter model and numerical solution are applied to solve the governing equation.

2. Theoretical model

2.1. Fundamental of modified strain gradient theory

Regarding the strain gradient theory modified and suggested by Lam et al. [17], $\bar{U}$ stored strain energy density in the linear elastic and isotropic material with small deformation is written as follows:

$$\bar{U} = \frac{1}{2}\left(\sigma_{ij}\varepsilon_{ij} + p_i\gamma_i + \tau^{(1)}_{ijk}\eta^{(1)}_{ijk} + m^s_{ij}\chi^s_{ij}\right) \tag{1}$$