Applied Mathematical Modelling 39 (2015) 2721–2739

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Electroelastic singularities and intensity factors for an interface crack in piezoelectric–elastic bimaterials

C.H. Xu [a], Z.H. Zhou [a,*], X.S. Xu [a], A.Y.T. Leung [b]

[a] State Key Laboratory of Structure Analysis of Industrial Equipment and Department of Engineering Mechanics, Dalian University of Technology, Dalian 116024, PR China

[b] Department of Civil and Architectural Engineering, City University of Hong Kong, Hong Kong

ARTICLE INFO

Article history:
Received 11 December 2013
Received in revised form 1 August 2014
Accepted 17 October 2014
Available online 23 November 2014

Keywords:
Symplectic approach
Piezoelectric–elastic bimaterials
Singularity stress/electric intensity factor
Interface crack

ABSTRACT

A symplectic approach based on the Hamiltonian system is proposed to analyze the electroelastic singularities and intensity factors of an interface crack in piezoelectric–elastic bimaterials. By introducing a total unknown vector consisting of generalized displacements and stresses, the Lagrangian equations are transformed to the Hamiltonian equations which can be solved by using the method of separation of variables. The total unknown vector can be expanded analytically in symplectic eigensolutions series (zero- and non-zero-eigensolutions). The unknown coefficients of the eigensolutions series are determined from the continuity conditions at the interface (electric conductor/insulation conditions) and outer boundary conditions. The study concludes that electroelastic singularities and intensity factors directly depend on the first few terms of non-zero-eigensolutions. Numerical examples for various conditions are given to show variations of singularity orders and intensity factors. These analyses may provide some guidance for the design of piezoelectric–elastic bimaterial system.

1. Introduction

In the engineering application, the electromechanical devices made of piezoelectric materials are usually bonded to an elastic material such as aluminum and epoxy. The mismatched property of the two dissimilar materials easily gives rise to cracks along bimaterial interfaces in service. Therefore, it is very necessary to explore the mechanism of interfacial fracture of piezoelectric–elastic devices.

A large number of investigations on this topic have been done and a great progress has been made. For the singularity analysis of piezoelectric–elastic bimaterials, Xu and Rajapakse [1] used Lekhnitskii's complex potential functions and Williams' eigenfunction expansion to examine the electro-elastic singularities at the corner of composite piezoelectric wedges. After that, Chue and Chen [2] further discussed the singularity of piezoelectric composite wedges for both in-plane and anti-plane problems. Chen et al. [3] developed a one-dimensional finite element formulation to determinate the order and angular variation of in-plane singular electro-elastic states in piezoelectric–elastic wedges and junctions.

Besides the singularity analysis above, the fracture mechanics of interface cracks in piezoelectric–elastic bimaterials have also received considerable attention. Tian and Chen [4], Ou and Chen [5,6], Li and Chen [7–9] studied the interface crack

* Corresponding author. Tel.: +86 13842853461; fax: +86 41184708393.
E-mail address: zhouzh@dlut.edu.cn (Z.H. Zhou).

http://dx.doi.org/10.1016/j.apm.2014.10.061
0307-904X/

problem in infinite piezoelectric–elastic bimaterials by using Stroth complex variable theory. Narita and Shindo [10,11], Narita et al. [12] reduced the electroelastic fracture problems to a set of singular integral equations which was obtained by the use of Fourier transforms. In the same manner, Kwon and Lee [13], Kwon and Lee [14], Li and Lee [15], Wang and Mai [16] studied the interface crack in three-layered structure constructed of a piezoelectric and two elastic materials; Wang and Meguid [17], Liu and Hsia [18] examined the coupled electromechanical behavior of a thin piezoceramic actuator embedded in or bonded to an elastic medium under in-plane mechanical and electrical loadings. The modified integral transform techniques were also widely used in this area, Ru [19] developed a hybrid complex-variable method which combined the Stroh's method of piezoelectric materials with the well-known Muskhelishvili's method of isotropic elastic materials to analyze the interface cracks between piezoelectric and isotropic elastic conductor/half-plane; Ishihara and Noda [20] employed the Bueckner's principle to study a two-dimensional electro-elastic problem of an infinite piezoelectric inhomogeneity, one of which contains a crack. In addition, Li et al. [21] proposed a semi-analytical technique which was based on the so-called scaled boundary finite element method to analyze two-dimensional cracks and notches inside piezoelectric composites. Akbarov and Yahnioglu [22] investigated the mechanisms of the buckling delamination for a sandwich plate-strip with piezoelectric face and elastic core layers by employing the finite elements method.

Because of the mathematical complications, less attention has been paid to the study of dynamic fracture mechanics of piezoelectric/piezomagnetic-elastic composites. Kwon and Lee [23] investigated the anti-plane transient response of a central crack normal to the interface between a piezoelectric ceramics and two same elastic materials by virtue of integral transform methods. Chen et al. [24] studied the problem of dynamic interfacial crack propagation in elastic–piezoelectric bimaterials subjected to uniformly distributed dynamic anti-plane loadings on crack faces by transform methods together with the Wiener–Hopf and Cagniard–de Hoop techniques. Zhang et al. [25] employed the integral transform technique to analyze the dynamic behavior of a piezoelectric–elastic laminate with a crack in the piezoelectric material under in-plane steady-state electro-mechanical loads. Hu and Chen [26] studied the moving interface crack between magnetoelectroelastic and functionally graded elastic layers under anti-plane mechanical loading and in-plane electric and magnetic loading, in which the Fourier transforms were applied.

However, it can be easily seen that these above-mentioned studies mainly focused on the infinite bodies and dealt with only cases of material combinations. And most of the works were within the framework of the semi-inverse solution method under the Lagrangian system and analytical solutions for piezoelectric composite problems were quite difficult to obtain directly. In order to overcome the problem, some researchers extended the symplectic expansion method, pioneered by Zhong and his associates [27,28], to apply to various branches of mechanics. Based on the symplectic expansion method under the Hamiltonian system, Zhong et al. [29], Liu and Li [30], Li et al. [31] derived the exact bending solutions of moderately thick rectangular plates; Zhang and Zhong [32] analyzed the singularity of multi-material junctions; Zhou et al. [33] obtained the complex stress intensity factors and T-stress at an edge bimaterial interface crack. After that, Zhou et al. [34] extended the method to the evaluation of stress and electric intensity factors in piezoelectric media. Wang and Qin [35] analyzed singularities near the apex of a multi-dissimilar piezoelectric wedge under anti-plane deformation within the symplectic framework.

In this paper, the symplectic expansion method is applied to the analysis of an interface crack in piezoelectric–elastic bimaterials. The elastic material considered in this study can be either conductor or composite. By introducing new components of stress and electric displacement, the problem is first reformulated in the Hamiltonian system. Then, in terms of the method of separation of variables, the Hamilton's canonical equations can be reduced to analyzing eigenvalues and their eigensolutions. Finally, the solutions of the problem are the linear combinations of eigensolutions of zero-eigenvalues and of non-zero-eigenvalues. The unknown coefficients are determined by the boundary conditions. The generalized stress and electric displacement intensity factors are directly obtained by the corresponding terms of the series. Numerical studies are conducted to illustrate the accuracy and validity of the proposed method.

2. Problem formulation

Consider a layered structure made by bonding together two different material elements, i.e. a piezoelectric material occupying the upper half plane and an elastic material occupying the lower half plane, as shown in Fig. 1. θ_1 and θ_2 are the boundary angles of the bimaterial. A set of polar cylindrical coordinates (r, θ, z) is established under the condition that the z-axis is along the longitudinal direction, and the origin is located at the crack tip. The poling direction of piezoelectric material is along the positive z-axis.

2.1. Governing equations

For the current problem, the out-of-plane displacement is coupled only with the in-plane electric fields such that

$$u_r^{(p)} = u_\theta^{(p)} = 0, \quad u_z^{(p)} = w^{(p)}(r, \theta), \tag{1a}$$

$$E_r = E_r(r, \theta), \quad E_\theta = E_\theta(r, \theta), \quad E_z = 0, \tag{1b}$$