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A newsvendor model with fuzzy price-dependent demand

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ABSTRACT

Price-dependence is an important characteristic for some inventory problems. This paper proposes a newsvendor model with fuzzy price-dependent demand, and discusses the conditions to determine the optimal pricing and inventory decisions jointly so that the expected profit could be maximized. Then an algorithm combining the method of ranking fuzzy numbers is developed to tackle the problem. Furthermore, comparison is made between the fuzzy model and the deterministic model to study the effect of the uncertain price-dependent demand, and the sensitivity properties of the joint optimal decisions are illustrated through numerical examples.

1. Introduction

The newsvendor model with price-dependent demand, which is established to make optimal inventory decisions and pricing decisions so that the expected profit could be maximized in a single period framework with uncertain demand, is one of the vast extensions of the classical newsvendor model. Petruzzi and Dada [1] give an extensive review on incorporating pricing into the newsvendor model, and provide results for additive (price influences the location of the demand distribution), multiplicative (price influences demand scale) demand cases in the single period problems and multiple period problems, respectively. Agrawal and Seshadri [2] extend the price-sensitive newsvendor model by taking utility function and risk tolerance into account. Yang [3] develops an integrated buyer–vendor model in the newsvendor framework for optimal pricing and inventory policy when demand is price sensitive, and studies the sensitivity of the total cost to the deterioration rate and the price-sensitive parameter. Ray [4] studies the effect of simultaneous price and attributes sensitivity of random demand in an integrated operations-marketing model. Gaur and Seshadri [5] address the problem of hedging inventory risk in the setting of newsvendor model which incorporate the price-correlated demand, and discuss the methods to construct optimal transactions that minimize the variance of profit and increase the utility for a risk-averse decision maker. Chen and Parlar [6] introduce a put option into the newsvendor model, in which a risk-averse firm could use the put option to maximize his expected utility function. Yao et al. [7] discuss how to obtain the optimal joint pricing and inventory decision for the newsvendor model under stochastic price-dependent demand without specific distribution functions. More recently, Kocabiyikoğlu and Popescu [8] offer a unifying perspective on the newsvendor with pricing problem by introducing a measure of elasticity of stochastic demand. Besides, more contributions to date for analyzing the price-dependent newsvendor problem could be found in Qin et al. review [9], which summarizes the recent literature on the newsvendor problem in the context of modeling customer demand, supplier costs, and the buyer risk profile.

However, due to the variety of the uncertainties and imprecision, traditional probability theory has its deficiency in certain cases. For instance, in absence of past statistical data, which is irrelevant for new products or the product with short lifecycles, it is hard to specify the distribution function of the sources of uncertainty, particularly for the uncertain demand. In addition, sometimes, the uncertainty of the demand is so variable that only linguistic terms could be employed to describe

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it. Hence, fuzzy theory, as a promising instrument, is used to describe and treat the uncertainty in these cases. Over the past decades, a small but suggestive literature on fuzzy newsvendor models has grown rapidly. Petrovi et al. [10] present two fuzzy newsvendor models to determine the optimal ordering quantity to minimize the total cost, and then discuss the effects of changing the membership function shapes of fuzzy inventory data on the optimal ordering quantity. Kao and Hsu [11] construct fuzzy newsvendor model and adopt a method for ranking fuzzy numbers to find optimal ordering quantity in terms of the cost. Ji and Shao [12] formulate a bilevel newsvendor model that integrate the behavior of the manufacturer and the retailers, and develop a hybrid programing algorithm based on genetic algorithm and fuzzy simulation to solve the hierarchical decision problem. Dutta et al. [13] propose a newsvendor model with reordering opportunities under fuzzy demand, and present a solution procedure using ordering of fuzzy numbers to determine the optimal ordering quantity in maximizing the expected resultant profit. Recently, Ryu and Yücesan [14] study three coordination policies by establishing a fuzzy newsvendor model, which is characterized with fuzzy demand, fuzzy wholesale price, and fuzzy market sales price. Su et al. [15] introduce a constructive approach combining the vector of fuzzy numbers to tackle the fuzzy newsvendor model. Yu and Jin [16] propose a fuzzy newsvendor model by extending the model of Kao and Hsu's [11], and study the return policy in a supply chain with symmetric channel information and asymmetric channel information respectively.

Despite of the various extensions of the fuzzy newsvendor models, none of the literature on fuzzy newsvendor models mentioned above considers the price dependent property of the demand though it is one of the classical properties of the demand in most practical cases. As a price-dependent demand could be affected by setting the selling price, and the essence of the price-setting newsvendor model is to control both the price and the inventory level to improve profits, our aim is to investigate the joint inventory and pricing policies for the newsvendor model with fuzzy price dependent demand. The model in this paper is in some way an extension of the Kao and Hsu's model [11] by incorporating price-dependent demand into it. Some of the issues to be addressed in this paper are:

(1) How to describe the fuzzy, price-dependent demand? How to solve the fuzzy newsvendor model? What are the joint inventory and pricing policies for the newsvendor model with fuzzy price dependent demand?
(2) How will the optimal decisions affected by the cost parameters? What are the effects of fuzziness and price-sensitivity of demand on the optimal decisions?

The remainder of the paper is organized as follows. Section 2 gives some preliminaries. In Section 3, a newsvendor model with fuzzy price dependent demand is proposed. In Section 4, a ranking method presented by Liou and Wang [17] is applied to solve the model. In Section 5, a simple algorithm is developed to determine the optimal price and inventory decisions to maximize the expected profit. In Section 6, the proposed model is compared with a deterministic model which ignores the uncertainty of the demand. In Section 7, numerical examples are given to illustrate the effectiveness of the model. Section 8 gives a conclusion.

2. Preliminaries

Definition 1 [17]. A real fuzzy number A is a fuzzy subset of the real line R with membership function f_A which possesses the following properties:

(a) f_A is a continuous function from R to a closed interval $[0,w], 0 \leqslant w \leqslant 1$;
(b) $f_A(x) = 0$, for all $x \in (-\infty, l]$;
(c) f_A is strictly increasing on $[l,m]$;
(d) $f_A(x) = 1$, for all $x \in [m,n]$;
(e) f_A is strictly decreasing on $[n,r]$;
(f) $f_A(x) = 0$, for all $x \in [r,\infty)$,

where l, m, n, r are real numbers, and A is convex, normal and bounded (i.e. $-\infty < l, r < \infty$).

Proposition 1 [17]. *Since $f_A^L : [l,m] \to [0,1]$ is continuous and strictly increasing, the inverse function of f_A^L, denoted by g_A^L, exists, and is also continuous and strictly increasing; similarly, since $f_A^R : [n,r] \to [0,1]$ is continuous and strictly decreasing, the inverse function of f_A^R, denoted by g_A^R, exists, and is also continuous and strictly decreasing. Both g_A^L and g_A^R are integrable.*

Definition 2 [17]. A is a fuzzy number with left membership function f_A^L and right membership f_A^R.Suppose that g_A^L is the inverse function of f_A^L, and g_A^R is the inverse function of f_A^R, then the left integral value of A is defined as $I_L(A) = \int_0^1 g_A^L(y)dy$, and the right integral value of A is defined as $I_R(A) = \int_0^1 g_A^R(y)dy$.

Definition 3 [17]. If A is a fuzzy number with membership f_A, defined as in Definition 1, then the total integral value with index of optimism ξ is defined as $I_T^{\xi}(A) = \xi I_R(A) + (1-\xi)I_L(A)$. Where $I_R(A)$ and $I_L(A)$ are the right and left integral values of A, respectively, and $\xi \in [0,1]$.