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Free magnetohydrodynamic flow and convection from a vertical spinning cone with cross-diffusion effects

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A B S T R A C T

A study of cross-diffusion effects on convection from a vertically spinning cone under the influence of an external magnetic field is considered. The non-linear two-point boundary value problem governing the flow is solved numerically. Two different types of surface heating, namely linear surface temperature (LST) and linear surface heat flux (LSHF) are considered. A parametric study addressing the effects of various flow parameters on the fluid properties, the skin friction, heat and mass transfer coefficients is given.

1. Introduction

Double-diffusive convection is an important fluid dynamics topic that involves convection driven by two different density gradients with different rates of diffusion [1]. A common example of double diffusive convection is in oceanography, where heat and salt concentrations exist with different gradients and diffuse at differing rates. The input of cold freshwater from an iceberg can affect both of these variables. Other examples can be found in crystal growing, convection in magma chambers (Huppert and Sparks [2]) and in the sun where temperature and Helium diffusions take place at differing rates. Double-diffusive free convection is also seen in sea-wind formations, where upward convection is also modified by Coriolis forces. This is of particular interest in oceans where the Earth's rotation plays a dominant role in many of the motions observed. In engineering applications, double diffusive convection is commonly visualized in the formation of microstructures during the cooling of molten metals, and fluid flows around shrouded heat-dissipation fins, and solar ponds.

Double-diffusive convection problems have been investigated by, among others, Nield [3], Baines and Gill [4], Guo et al. [5], Khanafer and Vafai [6], Sunil et al. [7] and Gaikwad et al. [8]. Studies have been carried out on horizontal, inclined and vertical surfaces in a porous medium by, among others, Cheng [9,10], Nield and Bejan [11], Ingham and Pop [12] and Pop and Ingham [13]. Na and Chiou [14] presented the problem of laminar natural convection in Newtonian fluids over the frustum of a cone. Natural convection over a vertical wavy cone has been investigated by Pop and Na [15]. Chamkha [16] investigated the coupled heat and mass transfer by natural convection of Newtonian fluids about a truncated cone in the presence of magnetic field and radiation effects and Yih [17] examined the effect of radiation in convective flow over a cone.

While heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of a more subtle nature. It is known that an energy flux can be generated by both temperature and composition gradients. The energy flux caused by a composition gradient is called the Dufour or diffusion-thermo effect. Mass fluxes cre-

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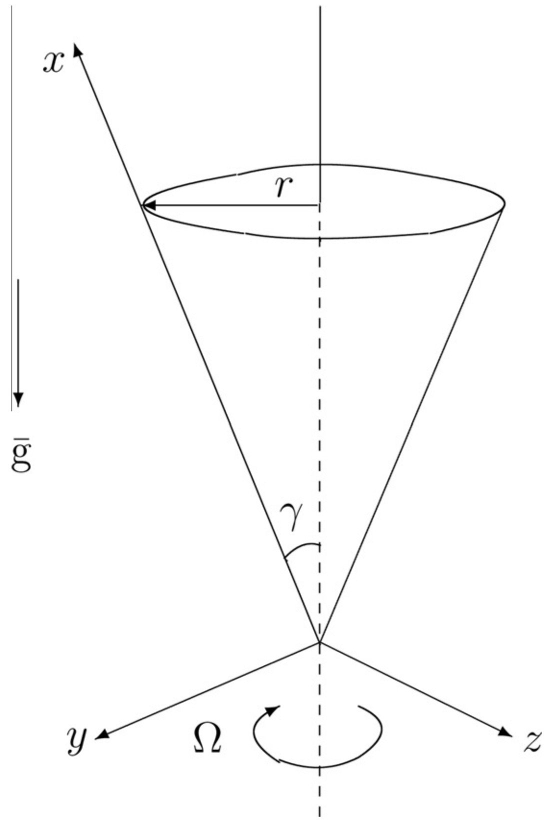

Fig. 1. Schematic sketch of a spinning vertical cone.

ated by temperature gradients give rise to the thermal-diffusion or Soret effect. Both effects have been extensively studied in gases, while the Soret effect has been studied both theoretically and experimentally in liquids, see Mortimer and Eyring [18]. It is generally accepted that the Dufour and the Soret effects are small compared to the effects described by Fourier and Fick's laws (Mojtabi and Charrier-Mojtabi [1]) and can therefore be neglected in many heat and mass-transfer processes. It has however been shown in a number of studies that there are exceptions in areas such as in geosciences when Dufour and Soret effects are significant and cannot be ignored, see for instance Kafoussias and Williams [19], Awad et al. [20] and the references therein.

Mortimer and Eyring [18] used an elementary transition state approach to obtain a simple model for Soret and Dufour effects in thermodynamically ideal mixtures of substances with molecules of nearly equal size. In their model the flow of heat in the Dufour effect was identified as the transport of the enthalpy change of activation as molecules diffuse. The results were found to fit the Onsager reciprocal relationship, Onsager [21]. Alam et al. [22] investigated the Dufour and Soret effects on steady combined free-forced convective and mass transfer flow past a semi-infinite vertical flat plate of hydrogen-air mixtures. They used the fourth order Runge–Kutta method to solve the governing equations of motion. Their study showed that the Dufour and Soret effects should not be neglected. Shateyi et al. [23] investigated the effects of diffusion-thermo and thermal-diffusion on MHD fluid flow over a permeable vertical plate in the presence of radiation and hall current. Awad and Sibanda [24] used the homotopy analysis method to study heat and mass transfer in a micropolar fluid subject to Dufour and Soret effects.

Lin and Lin [25] studied heat transfer from a rotating cone or disk to fluids for any Prandtl number. Himasekhar and Sarma [26] studied the effect of suction on heat transfer rates from a rotating cone while laminar mixed convection from a vertical rotating cone was studied by Himasekhar et al. [27]. Ece [28] studied free convection flow over a vertical spinning cone under a magnetic field using prescribe surface temperature and heat flux. Ece [29] has also considered free convection flow about a cone under mixed thermal boundary conditions and a magnetic field. Gorla and Nakamura [30] analyzed mixed convection of a micropolar fluid from a rotating cone by obtaining the non-similar solution of the problem.

There are many works relevant to the unsteady mixed convection flow due to rotating/fixed cones subjected to some external constraints. Osalusi et al. [31] studied the effect of combined viscous dissipation and Joule heating on unsteady mixed convection MHD flow on a rotating cone in a rotating fluid with variable properties in the presence of Hall and ion-slip currents. Takhar et al. [32] investigated unsteady mixed convection flow due to a rotating vertical cone with an