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An auxiliary parameter method using Adomian polynomials and Laplace transformation for nonlinear differential equations

Y. Khan [a,*], H. Vázquez-Leal [b], N. Faraz [c]

[a] Department of Mathematics, Zhejiang University, Hangzhou 310027, China
[b] University of Veracruz, School of Electronic Instrumentation, Xalapa, Mexico
[c] Modern Textile Institute, Donghua University, 1882 Yan'an Xilu Road, Shanghai 200051, China

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ABSTRACT

In this article, we proposed an auxiliary parameter method using Adomian polynomials and Laplace transformation for nonlinear differential equations. This method is called the Auxiliary Laplace Parameter Method (ALPM). The nonlinear terms can be easily handled by the use of Adomian polynomials. Comparison of the present solution is made with the existing solutions and excellent agreement is noted. The fact that the proposed technique solves nonlinear problems without any discretization or restrictive assumptions can be considered as a clear advantage of this algorithm over the numerical methods.

1. Introduction

The DDEs [1,2] play an important role in modeling complicated physical phenomena such as particle vibrations in lattices, currents flow in electrical networks, and pulses in biological chains. The solutions of these DDEs can provide numerical simulations of nonlinear partial differential equations, queuing problems, and discretizations in solid state and quantum physics. The study of discrete nonlinear systems governed by both ordinary and partial differential-difference (including lattice equations) and pure difference equations has drawn much attention in recent years particularly from the point of view of complete integrability [3].

Thus seeking solutions of nonlinear ordinary and partial differential equations still a significant problem that needs new techniques to develop exact and approximate solutions. Various powerful mathematical techniques such as Adomian decomposition method [4–8], homotopy perturbation method [9–12], variational iteration method [13–16], homotopy analysis method [17–19], homotopy perturbation transform method [20] and other classical methods [21–30] are used to obtained exact and approximate analytical solutions.

In this study, we present first time Auxiliary Laplace Parameter Method to solve the differential-difference equations. This method is developed from homotopy and Laplace method. We used homotopy method combined with the Laplace transform method for solving the famous differential-difference equations. The beauty of this proposed method is its capability of combining two powerful methods for obtaining fast convergence for any kind of ordinary or partial differential equation. We confirm, no such attempt has been made to use Auxiliary Laplace Parameter Method for solving well known differential-difference equations.

* Corresponding author.
E-mail address: yasir123@zju.edu.cn (Y. Khan).

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2. Auxiliary Laplace Parameter Method (ALPM)

To illustrate the basic idea of this method, we consider the nonlinear differential equation given by

$$Du(x,t) + Nu(x,t) - f(x,t) = 0, \tag{1}$$

with the following initial conditions:

$$u(x,0) = h(x), \quad u_t(x,0) = f(x), \tag{2}$$

where $f(x), h(x) \in C(R)$, D is the second order linear differential operator $D = \partial^2/\partial t^2$, N the general nonlinear differential operator and $f(x,t)$ the source term. First we apply the Laplace transform (denoted throughout this study by L) on both sides of Eq. (1):

$$L[Du(x,t)] + L[Nu(x,t)] - L[f(x,t)] = 0. \tag{3}$$

Using the differentiation property of Laplace transform, we have

$$s^2 L[u(x,t)] - sh(x) - f(x) + L[Nu(x,t) - f(x,t)] = 0, \tag{4}$$

$$L[u(x,t)] - \frac{h(x)}{s} - \frac{f(x)}{s^2} + \frac{1}{s^2} L[Nu(x,t) - f(x,t)] = 0. \tag{5}$$

The nonlinear term is decomposed as

$$Nu(x,t) = \sum_{n=0}^{\infty} A_n(u), \tag{6}$$

for some Adomian polynomials A_n (see [4–7]) that are given by

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[N\left(\sum_{i=0}^{\infty} \lambda^i u_i \right) \right]_{\lambda=0}, \quad n = 0,1,2,3,\ldots$$

We construct a following parameter depending homotopy by using homotopy technique:

$$H(v,t) = (1-t)L[v - u_0] + thL[N(v) - f(r)] = 0, \tag{7}$$

where $r \in R$, $t \in [0,1]$, L denotes the Laplace transform, h is non-zero auxiliary parameter $f(r)$, is a known function and u_0 is an initial approximation of equation.

The generalized form of Eq. (7) for Eq. (5), we have

$$L[v_n - u_0] + \frac{h}{s^2} L[N - f(r)] = 0, \tag{8}$$

Let us expand the unknown variable v in series as below

$$u = \lim_{n\to\infty} v = v_0 + v_1 + v_2 + \cdots \tag{9}$$

Operating with Laplace inverse on both sides of Eq. (8) gives

$$v_n(x,t) = I(x,t) - hL^{-1}\left[\frac{1}{s^2} L[A_n]\right], \tag{10}$$

$$v_0(x,t) = u(x,0) + tu_t(x,0) = I(x,t) \tag{11}$$

where $I(x,t)$ represents the term arising from the from the source term and prescribed initial condition.

$$v_{n+1}(x,t) = -hL^{-1}\left[\frac{1}{s^2} L[A_n]\right], \quad n \geqslant 0 \tag{12}$$

Substituting Eqs. (11) and (12) in Eq. (9) will give required result.

3. The differential-difference equations

Example 1. Consider the following differential-difference problem [2,13,14]:

$$\begin{aligned} &\frac{du_n}{dt} = u_n(u_{n+1} - u_{n-1}), \\ &u_n(0) = n. \end{aligned} \tag{13}$$

By applying the aforesaid method subject to the initial condition, we have