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H_∞ model reduction for port-controlled Hamiltonian systems

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ABSTRACT

This paper is concerned with the problem of H_∞ model reduction for the linear port-controlled Hamiltonian systems. The development includes both the continuous- and discrete-time cases. Some sufficient conditions are obtained for the existence of solutions in terms of linear matrix inequalities (LMIs) and a coupling non-convex rank constraint set. In addition, an explicit parametrization of the desired reduced-order model can be constructed if these conditions are satisfied. Furthermore, the conditions based on the strict LMIs without rank constraint are derived for the zeroth-order H_∞ approximation problem. Finally, the effectiveness of the proposed model reduction method is illustrated via a practical example.

1. Introduction

The port-controlled Hamiltonian system has been a popular research area in the last few decades [1–7]. Many research works are aimed at the energy dissipation, stability and passivity properties as well as the presence of conservation laws. Another important topic for the port-controlled Hamiltonian systems is to handle the interconnection of the physical system with other physical systems creating the so-called physical network. The dimensions of such the interconnected port-controlled Hamiltonian state-space systems with the lumped-parameter and distributed-parameter will rapidly grow in the practical applications. Therefore, an important issue is to deal with these high-dimensional port-controlled Hamiltonian systems for further analysis and control. In the control literature, there are a variety of techniques and methods used for model reduction serving port-controlled Hamiltonian systems. The passivity preserving model reduction is considered in [1]. The structure preserving model reduction of port-Hamiltonian systems is presented in [2–5]. The balancing techniques of port-Hamiltonian systems are given in [3,6].

On the other hand, the most commonly norm used for measuring model reduction error is the H_∞ norm. The H_∞ norm of the difference of the two systems is one of the most meaningful measures of the approximation error. The H_∞ model reduction problem, which consists of finding a low-order model $\hat{G}$ such that the H_∞ error norm of the transfer function of the error system is small, has received considerable attention in the literature [8–14]. Very recently, the linear matrix inequalities (LMIs) technique has been used to solve the model reduction problem for the different systems, including uncertain stochastic systems [8], singular systems [9,10], switched systems [11], Markovian jump systems [12] and bilinear systems [13], etc. It should be pointed out that, up to now, the model reduction problem for the linear port-controlled Hamiltonian systems based on the LMIs technique has received very little research attention.

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In this paper, we deal with the problem of H_∞ model reduction for the linear port-controlled Hamiltonian systems. The aim is to find a reduced-order model such that the H_∞ norm of the difference system between the original model and reduced order model is less than a given small constant. Both the continuous-time and the discrete-time port-controlled Hamiltonian systems are considered in this paper. In terms of certain LMIs and a coupling non-convex rank constraint set, some sufficient conditions for the solvability of this problem are obtained. It is worth pointing out that all the results obtained in this paper are derived without decomposing the original system matrices. Therefore, the desired reduced-order systems can be constructed directly.

The organization of the subsequent contents is as follows. In Section 2, the problem formulation and some preliminaries are given. The main results and their proofs are proposed in Section 3. In Section 4, a simulation example is given to demonstrate the effectiveness and applicability of the proposed results. Finally, the conclusions are presented in Section 5.

Throughout this paper, I represents the identity matrix with appropriate dimension. The notation of $X \geqslant Y (X > Y)$, where X and Y are real symmetric matrices, represents that $X - Y$ is a positive semi-definite (positive definite) matrix. We denote A^+ as the Moore–Penrose inverse of matrix A, and A_L, A_R are any full rank factors of A with $A_L A_R = A$. For a matrix $A \in R^{m\times n}$ with rank r, the orthogonal complement $A^\perp$ is defined as a (possibly non-unique) $(m - r) \times m$ matrix such that $A^\perp A = 0$. $\|A\|_\infty$ denotes the ∞ norm of matrix A.

2. Problem formulation and preliminaries

Consider the following input-state-output port-controlled Hamiltonian system [1,7] described by

$$\begin{cases} \dot{x}(t) = (J - R)Qx(t) + (B - P)u(t), \\ y(t) = (B^T + P^T)Qx(t) + (M + S)u(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $u(t) \in \mathbb{R}^m$ is the input vector; $y(t) \in \mathbb{R}^p$ is the output vector; $J = -J^T$ and $M = -M^T$ are the skew-symmetric matrices; $R = R^T \geqslant 0$ and $S = S^T \geqslant 0$ are the dissipation matrices; $Q = Q^T$ is the energy matrix; and the input matrix B and P specify the interconnection structure. Moreover the matrices R, S and P satisfy

$$\begin{bmatrix} R & P \\ P^T & S \end{bmatrix} \geqslant 0.$$

Let $M + S = D$ and $P = 0$, the port-controlled Hamiltonian system (1) can be reduced the following typical case

$$\Sigma_c : \begin{cases} \dot{x}(t) = (J - R)Qx(t) + Bu(t), \\ y(t) = B^T Qx(t) + Du(t). \end{cases} \tag{2}$$

It should be pointed out that many physical systems both in the electrical and mechanical fields can be described as the port-controlled Hamiltonian systems (2). The following example shows the port-controlled Hamiltonian representation of a ladder network in energy coordinates.

Example 1. Consider the n-dimensional linear ladder network shown in Fig. 1, with n being even; $C_1, C_2, \ldots, C_{n/2}$, $L_1, L_2, \ldots, L_{n/2}$, $R_1, R_2, \ldots, R_{n/2+1}$ being the capacitances, inductances and resistances of the corresponding capacitors, inductors and resistors, respectively, and $R_{n/2+1}$ the resistance of the load. The port-controlled Hamiltonian representation of this physical system is of the form (2) with

$$J = \begin{bmatrix} 0 & -1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & -1 \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}, \quad R = diag(0, R_1, \ldots, 0, R_{n/2} + R_{n/2+1}),$$

$$Q = diag(C_1^{-1}, L_1^{-1}, \ldots, C_{n/2}^{-1}, L_{n/2}^{-1}), \quad B^T = [1 \quad 0 \quad \cdots \quad 0 \quad 0], \quad D = 0.1.$$

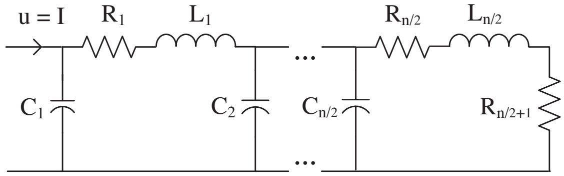

Fig. 1. n-Dimensional ladder network.