Applied Mathematical Modelling 37 (2013) 3021–3035

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Analysis of a conducting fluid in a thin annulus with rotating insulated walls under radial magnetic effect

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ARTICLE INFO

Article history:
Received 23 October 2011
Received in revised form 31 May 2012
Accepted 4 July 2012
Available online 17 August 2012

Keywords:
Electrically conducting fluid
Radial magnetic field
Couette flow
Hartmann number
Theoretical study

ABSTRACT

This work considers an electrically conducting fluid filled between two concentric cylindrical walls relatively close to each other. A theoretical solution for the steady Taylor–Couette flow between these two electrically insulated rotating cylinders under the influence of a radial magnetic field is provided in this work. By solving the appropriate set of governing equations simultaneously, the profiles of fluid tangential velocity component and induced magnetic field were obtained as complicated functions involving the modified Bessel functions of the first and second kinds of the first-order in terms of radial coordinates and Hartmann number. A computational study was also performed to validate the present theoretical solution. The analytical and computational results are identical when $Ha = 1$ while these results only slightly deviate from each other as Ha increases. Current results show that, the presence of the external magnetic field causes the flow close to the slower cylinder to accelerate while that close to the faster cylinder to decelerate. This has clearly implied the fact that an external magnetic field tends to make the velocity distribution across the inner and outer cylinders more uniform.

1. Introduction

In 1942, Hannes Olof Gösta Alfvén, a Swedish plasma physicist and Nobel laureate, had combined classical electromagnetism with fluid mechanics and developed the theory of magnetohydrodynamics [1]. Other than Alfvén, J. Hartmann was another pioneer who contributed a lot to the study of magnetohydrodynamics. He had experimentally and theoretically studied the flow of an electrically conducting incompressible non-Newtonian viscoelastic fluid between two parallel horizontal plates [2,3]. His theoretical derivations were capable of supporting the various phenomena he observed in his experiments. Based on his publications, it is discovered that there always exists in MHD problems a very important dimensionless parameter, the Hartmann number (Ha), whose square value is a measurement of the ratio of the electro-magnetic force to the fluid viscous force. This flow field Hartmann studied was then referred to as the Hartmann flow and had since then laid the foundation for future MHD investigation. If an external magnetic field is applied in the transverse direction, the magnitude of the flow velocity decreases with the strength of the external magnetic field because the external magnetic field generates the Lorentz force, which is an electromagnetic force acting on the conducting fluid. Not only so, the flow pattern also depends heavily on the wall conductivity. The velocity gradients on perfectly insulated walls are always the same irrespective the value of Ha. Also, the maximum velocity between a pair of insulated walls is always greater than that between perfectly conducting walls. On the other hand, when an external magnetic field is applied on the classical Couette flow perpendicular to the infinite planes, the flow pattern of the conducting fluid filling the plates bares some fundamental nature

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0307-904X/$ - see front matter
http://dx.doi.org/10.1016/j.apm.2012.07.011

Nomenclature

$\vec{B}$	magnetic flux density, Tesla
B_0	external magnetic field density, Tesla
b_θ	angular component of induced magnetic field density, Tesla
$\underline{D}$	electric flux density, C/m^2
$\underline{E}$	electric intensity, V/m
$\vec{g}$	gravitational acceleration, m^2/s
H	magnetic intensity, A/m
Ha	Hartmann number, –
j	induced current density, A/m^2
p	pressure, Pa
q	electric charge density, C/m^3
r	radial coordinate, m
t	time, s
V	fluid velocity, m/s
υ_0	an arbitrary velocity scale used in normalization, m/s
υ_θ	fluid angular velocity component, m/s

Greek

ε	permittivity of material, F/m
μ	permeability of material, H/m
ν	fluid kinematic viscosity, m^2/s
θ	angular coordinate, –
ρ	fluid density, kg/m^3
σ	electric conductivity, 1/Ω m
ω	rotational speed, 1/s

Subscripts

i	inner cylinder
o	outer cylinder
θ	angular direction

of the Hartmann flow and is named the MHD Couette flow. When both planes are perfectly insulated, the half of the flow field next to the fast moving plane decelerates while the other half accelerates. The velocity at the location halfway between the planes is actually the average of the velocities at the upper and lower planes. If the planes are perfectly conducting, the flow field everywhere decelerates with *Ha* [4].

Shercliff [5,6] studied the flows of a conducting fluid from the flowmetry point of view and solved the problem mathematically for a rectangular pipe. Hunt and Stewartson [7] studied the flow in rectangular pipes under a uniform transverse magnetic field for various types of walls and various sections. Their solution for high Hartmann numbers involved an integral equation of a standard form. Gupta and Singh [8,9] obtained the exact solutions of some steady and unsteady flow problems in pipes of different sections. Considering a different geometrical shape, Heiser and Shercliff [10] developed the theory for the motion of a viscous conducting liquid filled within a pair of rotating long concentric cylinders subjected to a radial magnetic field corresponding to high Hartmann numbers. They have also performed experiments using mercury and confirmed their theory. Later, Molokov and Allen [11] extended the theory Heiser and Shercliff developed. Under the assumption of a high Hartmann number, the set of equations they encountered were successfully decoupled.

In 1999, Bessaih et al. [12] made use of the computational approach to study the flow field of a MHD liquid metal filled in a cylindrical enclosure driven by two rotating horizontal disks. In this work, the vertical walls were either perfectly conducting or perfectly insulated. Their results showed that the primary flow characteristics could be effectively controlled through a good choice of the electric conductivities of the disk and cylinder walls. Later, Witkowski and Walker [13] numerically studied the steady liquid metal flow between a pair of insulated cylinders subjected to a rotational motion and a uniform weak transverse magnetic field. The flow pattern mainly consisted of an axisymmetric part combined with a weak nonaxisymmetric part. The axisymmetric flow they discovered was a rigid-body rotation similar to the classical Ekman flow. In 2004, Hayat et al. [14] studied the influence of magnetic field on an Oldroyd 6-constant magnetohydrodynamic fluid, a non-Newtonian fluid. They employed the Homotopy Analysis Method (HAM) to obtain the analytical solution corresponding to (a) a steady Couette flow, (b) a steady Poiseuille flow, and (c) a more generalized steady Couette flow. Their solution showed a strong dependence of the fluid non-Newtonian parameters. In another publication, Hayat et al. [15] also considered two infinitely extended non-conducting parallel plates between which the same fluid flowed under the influence of a uniform transverse magnetic field. Different from the upper boundary condition investigated in [14], the upper plate considered in [15] oscillated while the lower plate was at rest. Their analytical solution again suggested that the magnetic flow field