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Analytical solution of piezolaminated rectangular plates with arbitrary clamped/simply-supported boundary conditions under thermo-electro-mechanical loadings

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ABSTRACT

This paper extends an analytical method for static analysis of general cross-ply piezolaminated rectangular plates with any combination of clamped/simply-supported boundary conditions under uncoupled thermo-electro-mechanical loadings. This method is based on the novel superposition method and the first-order shear deformation theory (FSDT). The FSDT enables this expanded method to consider the effect of shear deformation of the plate. The process of applying electrical and thermal resultants causes some advantages due to its simplicity and less computational process. In this analysis displacement components are written in terms of unknown force and moment resultants. Using Fourier series for displacement components, mechanical, thermal, and/or electrical stress resultants, the complex governing differential equations of the plate are reduced to a set of linear algebraic equations with non-trivial solution. The obtained equations may be solved analytically to determine the unknown stress resultants. Several examples are proposed, and their obtained numerical results are compared with those available in the literature to verify the convergence, high accuracy, and the capability of the present method to analyze the static behavior of piezolaminated plates. It is found that there is high agreement between the present results with those obtained by other investigators.

1. Introduction

Since the late 1890s, the importance of piezoelectric materials has steadily increased. Piezoelectric elements can be used in static applications such as torsion of helicopter blades, deflection of missile fins, airfoil shape changes, or in dynamic applications such as structural vibration, acoustical generated noise, etc. A great enhancement of piezoelectric materials has been given by utilizing advanced fiber reinforced composites, in which piezoelectric materials are embedded or surface-mounted in structures. Hence, the study of the hybrid piezoelectric laminated plates has received considerable attention during the recent decades.

There are several exact solutions for static analysis of hybrid piezoelectric laminates that are limited to simply-supported edges. Wang and Rogers [1] developed analytical solutions based on the classical laminated plate theory (CLPT) for simply-supported plates with surface-bonded or embedded piezoelectric layers. Mitchell and Reddy [2], using higher-order shear deformation theory (HSDT), studied mechanical displacement of simply-supported piezoelectric rectangular hybrid laminates. Lee and Moon [3] used Kirchhoff plate theory to develop a piezoelectric laminate theory of laminated plates with distributed piezoelectric layers to exhibit both bending and torsion deformations of the considered plate under an applied

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electric field. In addition, Lee [4] developed his previous study to control and sense of bending, torsion, shearing, shrinking, and stretching of a flexible piezolaminated plate.

Cheng et al. [5] utilized the assumption of the simple higher-order shear deformation theory to analyze laminated composite plates integrated with the random poled piezoelectric layers. They obtained the fundamental equations of piezoelectric/composite anisotropic laminate, based on the generalized Hamilton's variation principle. Furthermore, they used the von Karman strains to account for the geometric nonlinear effect of the practical larger-amplitude deflection on the electro-elastic behavior of smart composite structures.

Ray et al. [6] presented an analytical solution based on the approach of Pagano [7] for simply-supported square composite laminate with surface bonded piezoelectric layers. Jonnalagadda et al. [8] employed the first-order shear deformation theory to analyze analytically the static behavior of simply-supported piezothermoelastic hybrid plates under thermo-electro-elastic loading. Furthermore, they established a nine-node Lagrangian finite element formulation for analysis of clamped/simply-supported hybrid plates.

However, owing to the complexity of the governing equations in non-simply-supported plates only few analytical and semi-analytical studies have concentrated on the piezolaminated composite plates with various boundary conditions. The Levy-type solution for the bending of cross-ply hybrid rectangular plate with at least two opposite simply-supported edges based on the CLPT and FSDT was presented by Kapuria et al. [9]. Vel and Batra [10–12] provided an analytical solution to study the generalized plane strain deformations of piezoelectric laminated plates with arbitrary boundary condition via Eshelby–Stroh formulism. Guanghui et al. [13] presented a semi-analytical method for clamped plates with piezoelectric patches based on modified mixed variational principal. Zhang and Sun [14] developed the Rayleigh–Ritz formulation based on the principle of stationary potential energy for a clamped–clamped-free-free square sandwich plate with a piezoelectric core. Edery-Azulay and Abramovich [15] also presented a model based on the Levy method and FSDT for piezolaminaled plate with at least two opposite simply-supported edges. They used approximate Rayleigh–Ritz method to analyze a plate with piezoelectric patches under different boundary conditions. Edery-Azulay and Abramovich [16] employed semi-analytically extended Kantorovich iterative procedure based on the CLPT for piezoelectric plates with various boundary conditions. The electromagnetothermoelastic behaviors of a hollow cylinder composed of functionally graded piezoelectric material (FGPM) were studied analytically by Dai et al. [17]. They used infinitesimal theory to investigate effect of gradient parameter variation of the FGPM on the elastic displacement, stresses and elastic potential.

The superposition method has been used in some studies for isotropic, laminated composite, and vibration of functionally graded material (FGM) plates. Employing the Levy solution and method of superposition, Timoshenko and Krieger [18] studied deflection and bending moments for rectangular isotropic thin plates, each of them would involve violation of either the governing equation and some of the boundary conditions. Yu et al. [19] utilized the superposition method in order to achieve solutions for the problems with clamped/simply-supported edges. Gorman and Ding [20–21] and Gorman [22] extended an analytical procedure involved the superposition method to study the vibration of rectangular plates. Bhaskar and Kaushik [23] presented a novel superposition method for symmetric cross-ply laminated composite plates with clamped/simply-supported edges using the CLPT. In addition, they [24] developed their previous study for unsymmetric cross-ply laminated composite plates with clamped edges. Umasree and Bhaskar [25] used superposition method for symmetric cross-ply clamped laminated plates based on zig–zag type higher-order theory.

The purpose of the present study is to develop an analytical method for bending analysis of rectangular hybrid general cross-ply piezolaminated plates with any arbitrary clamped/simply boundary conditions under thermo-electro-mechanical loading. The thermal and electrical resultants are introduced and it is shown how these resultants must be added to the formulations for obtaining analytical solutions. It is well established that the CLPT is inadequate for the analysis of moderately thick plates because it does not consider the effect of shear deformation of the plate. This effect is much considerable for composite laminated plates in which the ratio of elastic modulus to shear modulus is high. This paper carried out its formulation based on the FSDT in order to consider the influence of the transverse shear effect on the deformation of peizolaminated rectangular plates under thermo-electro-mechanical loadings. It is shown that the method provides accurate predictions for all displacements with good convergence, however, the convergence rate can be dependent on the type of lamination, loading, and boundary conditions. The high accuracy of the present analytical method is verified with comparing the obtained numerical results with those obtained by other investigators.

2. Fundamental equations

The geometry and coordinate used in the following sections are shown in Fig. 1. The piezolaminated plate has a total thickness h, length a in the longitudinal (x–) direction, and width b in the lateral (y–) direction. It is assumed that the middle plane of the plate lies on the x–y plane of a Cartesian coordinate system. Some of the layers can be piezoelectric of mm2 class with poling in direction z, perpendicular to the plate. According to the FSDT, the displacement field can be expressed as

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\psi(x,y), \\ v(x,y,z) &= v_0(x,y) + z\varphi(x,y), \\ w(x,y,z) &= w_0(x,y), \end{aligned} \tag{1}$$