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A new sinusoidal shear deformation theory for bending, buckling, and vibration of functionally graded plates

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ABSTRACT

A new sinusoidal shear deformation theory is developed for bending, buckling, and vibration of functionally graded plates. The theory accounts for sinusoidal distribution of transverse shear stress, and satisfies the free transverse shear stress conditions on the top and bottom surfaces of the plate without using shear correction factor. Unlike the conventional sinusoidal shear deformation theory, the proposed sinusoidal shear deformation theory contains only four unknowns and has strong similarities with classical plate theory in many aspects such as equations of motion, boundary conditions, and stress resultant expressions. The material properties of plate are assumed to vary according to power law distribution of the volume fraction of the constituents. Equations of motion are derived from the Hamilton's principle. The closed-form solutions of simply supported plates are obtained and the results are compared with those of first-order shear deformation theory and higher-order shear deformation theory. It can be concluded that the proposed theory is accurate and efficient in predicting the bending, buckling, and vibration responses of functionally graded plates.

1. Introduction

Functionally graded materials (FGMs) are a class of composites that have continuous variation of material properties from one surface to another and thus eliminate the stress concentration found in laminated composites. FGMs are widely used in many structural applications such as mechanics, civil engineering, aerospace, nuclear, and automotive. In company with the increase in the application of FGM in engineering structures, many computational models have been developed for predicting the response of functionally graded (FG) plates. These models can either be developed using displacement-based theories (when the principle of virtual work is used) or displacement-stress-based theories (when Reissner's mixed variational theorem is used). In general, these theories can be classified into three main categories: classical plate theory (CPT); first-order shear deformation theory (FSDT); and higher-order shear deformation theory (HSDT).

The CPT, which neglects the transverse shear deformation effects, provides accurate results for thin plates [1–4]. For moderately thick plates, it underestimates deflections and overestimates buckling loads and natural frequencies. The FSDT accounts for the transverse shear deformation effect, but requires a shear correction factor to satisfy the free transverse shear stress conditions on the top and bottom surfaces of the plate [5–11]. Although the FSDT provides a sufficiently accurate description of response for thin to moderately thick plates, it is not convenient to use due to difficulty in determination of correct value of the shear correction factor. To avoid the use of shear correction factor, many HSDTs were developed based on

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the assumption of quadratic, cubic or higher-order variations of in-plane displacements through the plate thickness, notable among them are Reddy [12], Karama et al. [13], Zenkour [14–16], Xiao et al. [17], Matsunaga [18], Pradyumna and Bandyopadhyay [19], Fares et al. [20], Talha and Singh [21–22], Benyoucef et al. [23], Atmane et al. [24], Meiche et al. [25], Mantari et al. [26], and Xiang et al. [27]. Among the aforementioned HSDTs, the well-known HSDTs with five unknowns include: the Reddy's theory [12], the sinusoidal shear deformation theory [14–16], the hyperbolic shear deformation theory [23–24], the exponential shear deformation theory [13,26]. Although the HSDTs with five unknowns are sufficiently accurate to predict response of thin to thick plate, their equations of motion are much more complicated than those of FSDT and CPT. Therefore, there is a scope to develop a HSDT which is simple to use.

This paper aims to develop a simple sinusoidal shear deformation theory for bending, buckling, and vibration analyses of FG plates. This theory is based on assumption that the in-plane and transverse displacements consist of bending and shear parts. Unlike the conventional sinusoidal shear deformation theory [14–16], the proposed sinusoidal shear deformation theory contains four unknowns and has strong similarities with CPT in many aspects such as equations of motion, boundary conditions, and stress resultant expressions. Material properties of FG plate are assumed to vary according to power law distribution of the volume fraction of the constituents. Equations of motion are derived from the Hamilton's principle. The closed-form solutions are obtained for simply supported plates. Numerical examples are presented to verify the accuracy of the proposed theory in predicting the bending, buckling, and vibration responses of FG plates. It should be pointed out that Merdaci et al. [28], Ameur et al. [29], and Tounsi et al. [30] recently developed a theory that is similar with the present one. However, their works are limited to only bending problems.

2. Theoretical formulations

2.1. Basic assumptions

The assumptions of the present theory are as follows:

i. The displacements are small in comparison with the plate thickness and, therefore, strains involved are infinitesimal.
ii. The transverse normal stress σ_z is negligible in comparison with in-plane stresses σ_x and σ_y.
iii. The transverse displacement u_3 includes two components of bending w_b and shear w_s. These components are functions of coordinates x, y, and time t only.

$$u_3(x,y,z,t) = w_b(x,y,t) + w_s(x,y,t). \tag{1}$$

iv. The in-plane displacements u_1 and u_2 consist of extension, bending, and shear components.
$u_1 = u + u_b + u_s$ and $u_2 = v + v_b + v_s$. (2)

- The bending components u_b and v_b are assumed to be similar to the displacements given by the classical plate theory. Therefore, the expressions for u_b and v_b are

$$u_b = -z\frac{\partial w_b}{\partial x} \text{ and } v_b = -z\frac{\partial w_b}{\partial y}. \tag{3a}$$

- The shear components u_s and v_s give rise, in conjunction with w_s, to the sinusoidal variations of shear strains γ_{xz}, γ_{yz} and hence to shear stresses σ_{xz}, σ_{yz} through the thickness h of the plate in such a way that shear stresses σ_{xz}, σ_{yz} are zero at the top and bottom surfaces of the plate. Consequently, the expression for u_s and v_s can be given as

$$u_s = -\left(z - \frac{h}{\pi}\sin\frac{\pi z}{h}\right)\frac{\partial w_s}{\partial x} \text{ and } v_s = -\left(z - \frac{h}{\pi}\sin\frac{\pi z}{h}\right)\frac{\partial w_s}{\partial y}. \tag{3b}$$

2.2. Kinematics

Based on the assumptions made in the preceding section, the displacement field can be obtained using Eqs. (1)-(3) as

$$\begin{aligned} u_1(x,y,z,t) &= u(x,y,t) - z\frac{\partial w_b}{\partial x} - f\frac{\partial w_s}{\partial x}, \\ u_2(x,y,z,t) &= v(x,y,t) - z\frac{\partial w_b}{\partial y} - f\frac{\partial w_s}{\partial y}, \\ u_3(x,y,z,t) &= w_b(x,y,t) + w_s(x,y,t), \end{aligned} \tag{4}$$

where

$$f = z - \frac{h}{\pi}\sin\frac{\pi z}{h}. \tag{5}$$