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A grey relational projection method for multi-attribute decision making based on intuitionistic trapezoidal fuzzy number

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ABSTRACT

With respect to multiple attribute decision making (MADM) problems in which the attribute value takes the form of intuitionistic trapezoidal fuzzy number, and the attribute weight is unknown, a new decision making analysis methods are developed. Firstly, some operational laws and expected values of intuitionistic trapezoidal fuzzy numbers, and distance between two intuitionistic trapezoidal fuzzy numbers, are introduced. Then information entropy method is used to determine the attribute weight, and the grey relational projection method combined grey relational analysis method and projection method is proposed, and to rank the alternatives are done by the relative closeness to PIS which combines grey relational projection values from the positive ideal solution and negative ideal solution to each alternative. Finally, an illustrative example is given to verify the developed approach and to demonstrate its practicality and effectiveness.

1. Introduction

Multiple attribute decision making (MADM) is an important part of modern decision science. Many achievements have been made in research on MADM problems based on the various attribute values, such as interval number, triangular fuzzy number, and trapezoidal fuzzy number etc. [1–8]. Since the fuzzy set theory is proposed by Zadeh [9], it has been a very good tool to research the Fuzzy MADM problems. However, the fuzzy set is used to character the fuzziness just by membership degree. Different from fuzzy set, there is another parameter: no-membership degree in intuitionistic fuzzy set which is proposed by Atanassov [10,11]. Obviously, the intuitionistic fuzzy set (IFS) can describe and character the fuzzy essence of the objective world more exquisitely [10], and it has received more and more attention since its appearance. Later, Atanassov and Gargov [12], Atanassov [13] further introduced the interval-valued intuitionistic fuzzy set (IVIFS), which is a generalization of the IFS. The fundamental characteristic of the IVIFS is that the values of its membership function and non-membership function are intervals rather than crisp numbers. Xu [14] investigated the interval-valued intuitionistic fuzzy MADM with the information about attribute weights are incompletely known or completely unknown, a method based on the ideal solution was proposed. Wang [15] investigated the interval-valued intuitionistic fuzzy MADM with incompletely known weight information. A nonlinear programming model is developed, and ranking is performed through the comparison of the distances between the alternatives and idea/anti-idea alternative. Shu et al. [16] gave the definition and operational laws of intuitionistic triangular fuzzy number and proposed an algorithm of the intuitionistic fuzzy fault-tree analysis. Zhang and Liu [17] used the triangular fuzzy number to denote the membership degree and the non-membership degree and proposed the triangular intuitionistic fuzzy number, then the weighted arithmetic averaging operator and the weighted arithmetic average operator was defined, and an approach to multiple attribute group

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decision making (MAGDM) with triangular intuitionistic fuzzy information was developed. Wang [18] gave the definition of intuitionistic trapezoidal fuzzy number and interval intuitionistic trapezoidal fuzzy number. Wang and Zhang [19] gave the definition of expected values of intuitionistic trapezoidal fuzzy number and proposed the programming method of multi-criteria decision-making based on intuitionistic trapezoidal fuzzy number with incomplete certain information. Wang and Zhang [20] developed the Hamming distance of intuitionistic trapezoidal fuzzy numbers and intuitionistic trapezoidal fuzzy weighted arithmetic averaging (ITFWAA) operator, then proposed multi-criteria decision-making method with incomplete certain information based on intuitionistic trapezoidal fuzzy number. Wang and Zhang [21] proposed some aggregation operators, including intuitionistic trapezoidal fuzzy weighted arithmetic averaging operator and weighted geometric averaging operator, and defined expected values of intuitionistic trapezoidal fuzzy number. Based on expected values, score function and accuracy function of intuitionitsic trapezoidal fuzzy numbers were defined, and a ranking of the whole alternative set can be attained. Wan and Dong [22] proposed the expectation and expectant score by the coordinates of gravity center of intuitionistic trapezoidal fuzzy number, and defined ordered weighted aggregation operator and hybrid aggregation operator for intuitionistic trapezoidal fuzzy numbers. Finally, the results of group decision making were presented according to the expectation and expectant score. Wei [23] proposed some aggregation operators, including intuitionistic trapezoidal fuzzy ordered weighted averaging (ITFOWA) operator and intuitionistic trapezoidal fuzzy hybrid aggregation (ITFHA) operator, and developed an approach to multiple attribute group decision making (MAGDM) with intuitionistic trapezoidal fuzzy information. Du and Liu [24] proposed an extended fuzzy VIKOR method with respect to multiple attribute decision making problems in which the attribute value takes the form of intuitionistic trapezoidal fuzzy number, and the attribute weight is unknown.

Obviously, the study on the multiple attribute decision making problems with intuitionistic trapezoidal fuzzy information has just started. The aim of this paper is to propose an approach for the MADM problems in which the attribute value takes the form of intuitionistic trapezoidal fuzzy number, and the attribute weight is unknown. Firstly, some operational laws, expected values and distance of intuitionistic trapezoidal fuzzy number are introduced. Then information entropy method is used to determine the attribute weight, and the grey relational projection method combined grey relational analysis method and projection method is proposed to rank the alternatives. Finally, an illustrative example is given to verify the developed approach.

2. Preliminaries

In the following, we shall introduce some basic concepts related to intuitionistic trapezoidal fuzzy numbers.

Definition 1 (*[18–21]*). Let $\tilde{a}$ be an intuitionistic trapezoidal fuzzy numbers, its membership function is defined as

$$\mu_{\tilde{a}}(x)=\begin{cases}\frac{x-a}{b-a}\mu_{\tilde{a}}, & a\leqslant x<b,\\ \mu_{\tilde{a}}, & b\leqslant x\leqslant c,\\ \frac{d-x}{d-c}\mu_{\tilde{a}}, & c<x\leqslant d,\\ 0, & \text{otherwise}.\end{cases} \tag{1}$$

Its non-membership function is defined as

$$v_{\tilde{a}}(x)=\begin{cases}\frac{(b-x)+v_{\tilde{a}}(x-a_1)}{b-a_1}, & a_1\leqslant x<b,\\ v_{\tilde{a}}, & b\leqslant x\leqslant c,\\ \frac{(x-c)+v_{\tilde{a}}(d_1-x)}{d_1-c}, & c<x\leqslant d_1,\\ 0, & \text{otherwise},\end{cases} \tag{2}$$

where $0\leqslant\mu_{\tilde{a}}\leqslant 1$, $0\leqslant v_{\tilde{a}}\leqslant 1$ and $0\leqslant\mu_{\tilde{a}}+v_{\tilde{a}}\leqslant 1$; $a,a_1,b,c,d,d_1\in R$. The intuitionistic fuzzy number is denoted as $\tilde{a}=\langle([a,b,c,d];\mu_{\tilde{a}}),([a_1,b,c,d_1];v_{\tilde{a}})\rangle$. Different from fuzzy numbers, the intuitionistic fuzzy numbers have another parameter: non-membership function, which is used to express the extent to which the decision makers think that the element does not belong to $[a_1,b,c,d_1]$. When $\mu_{\tilde{a}}=1$, $v_{\tilde{a}}=0$, $\tilde{a}$ is changed into trapezoidal fuzzy number. Generally, there is $[a,b,c,d]=[a_1,b,c,d_1]$ in intuitionistic trapezoidal fuzzy number $\tilde{a}$, here, denoted as $\tilde{a}=([a,b,c,d];\mu_{\tilde{a}},v_{\tilde{a}})$.

Definition 2 (*[18–21]*). Let $\tilde{a}_1=([a_1,b_1,c_1,d_1];\mu_{\tilde{a}_1},v_{\tilde{a}_1})$ and $\tilde{a}_2=([a_2,b_2,c_2,d_2];\mu_{\tilde{a}_2},v_{\tilde{a}_2})$ be two intuitionistic trapezoidal fuzzy numbers, and $\lambda\geqslant 0$, then

(1) $$\tilde{a}_1+\tilde{a}_2=([a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2];\mu_{\tilde{a}_1}+\mu_{\tilde{a}_2}-\mu_{\tilde{a}_1}\mu_{\tilde{a}_2},v_{\tilde{a}_1}v_{\tilde{a}_2}), \tag{3}$$

(2) $$\tilde{a}_1\otimes\tilde{a}_2=([a_1a_2,b_1b_2,c_1c_2,d_1d_2];\mu_{\tilde{a}_1}\mu_{\tilde{a}_2},v_{\tilde{a}_1}+v_{\tilde{a}_2}-v_{\tilde{a}_1}v_{\tilde{a}_2}), \tag{4}$$