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An analytical solution for dynamic behavior of a beam–column frame with a tip body

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ABSTRACT

This paper analyzes the vibration characteristics of a beam-column frame, typical examples of which are often found in optical pickup actuators of optical disc drives (ODDs) and many architectural structures. The dynamic behaviour of this beam structure is predicted by solving mathematically its vibration characteristics governed by beam configurations. For practical applications and simplicity in the analysis, the vibration analysis for the structure is limited to lateral and longitudinal directions of the beams. As a result, mode and modal frequencies are obtained from mathematical expressions. The accuracy of vibration characteristics, which is mathematically induced, is demonstrated by a finite element (FE) analysis. Finally, it is shown that mode shapes are modified by using design values with the mathematical expressions.

1. Introduction

Many authors contributed to the studies which treated the vibration characteristics of a beam with a tip body. Laura et al. showed an analytical solution for the lateral vibration of single cross-sectional cantilever with a tip body [1]. Bhat et al. introduced the analytical solution for the lateral vibration of a uniform cantilevered beam with a tip body slender in the axial direction [2]. Rossi et al. obtained natural frequencies in a non-uniform cantilevered beam with a tip mass [3]. Auciello obtained the solution of a vibration problem in a linearly tapered cantilever beam with tip mass of rotary inertia and eccentricity [4]. Gokdag et al. presented an exact procedure to obtain natural frequencies and mode shapes of a system consisting of a beam with monosymmetric open cross section carrying a tip body and springs at one end [5]. Dokumaci presented natural frequencies and modes affected by the coupling of bending and torsional vibrations [6]. Farghaly presented the vibration of an axially loaded cantilever beam with an elastically mounted end mass [7]. Banerjee derived exact frequency equation and mode shape expressions for a coupled bending-torsional beam with cantilever end condition [8].

Nowadays, more complicated systems such as multi-beam structures are used in various mechanical equipments. Nevertheless, researches on vibration analysis of such structures are not plentiful. Lee mathematically analyzed the vibration characteristics of four parallel and uniform beams joined by a tip body at their free ends and derived modal frequencies and shapes of an optical pickup actuator [9]. That paper showed the vibration characteristics of uniform four beams with a tip body, which were related to a pure-axial vibration, a coupled bending-torsional vibration, and two coupled axial-bending vibrations. Based on the same approach of [9] which shows analytical derivation of the equations of motions and also boundary conditions in a flexible system, this paper aims to mathematically analyze and improve the vibration characteristics of a multi-beam structure joined by a tip body, which are related to a coupled axial-bending vibration of a

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Nomenclature

Beam
s number of beam
l length of beam
A_s cross-sectional area of sth beam
I_s areal moment of inertia of sth beam
ρ_s density of sth beam
E_s Young's modulus of elasticity of sth beam

Tip body
m mass of tip body
a,c distances between the mass center of tip body and the free end of the 1st beam along the x_1 and x_2 axes, respectively
$J = mk^2$ mass moment of inertia of tip body about the axis through the mass center, perpendicular to x_1–x_2 plane
k radius of gyration about the axis through the mass center, perpendicular to x_1–x_2 plane
b_{s-1} distance between mid-lines of 1st and sth beams

Motion
x_1, x_2, x_3 Cartesian coordinates
t time
$u_s(x_1,t)$ axial deformation of sth beam in x_1 direction
$v_s(x_1,t)$ lateral deflection of sth beam in x_2 direction
$u'_s(x_1,t) = \partial u_s(x_1,t)/\partial x_1$ extensional strain of sth beam
$v'_s(x_1,t) = \partial v_s(x_1,t)/\partial x_1$ lateral deflection angle of sth beam in x_2 direction, derivative of $v_s(x_1,t)$ with respect to x_1
$\dot{u}_s(x_1,t) = \partial u_s(x_1,t)/\partial t$ derivative of $u_s(x_1,t)$ with respect to t
$\dot{v}_s(x_1,t) = \partial v_s(x_1,t)/\partial t$ derivative of $v_s(x_1,t)$ with respect to t
ω radian natural frequency

Expression normalised by time constant σ and beam length l
$\sigma = \sqrt{\rho_1 l^2/E_1}$ time-normalising factor
$\bar{x}_1 = x_1/l$ non-dimensional coordinate
$\bar{t} = t/\sigma$ non-dimensional time
$\bar{a} = a/l, \bar{c} = c/l$ normalised distances between the mass center of tip body and the free end of the 1st beam along the x_1 and x_2 axes, respectively
$\bar{k} = k/l$ normalised radius of gyration of tip body about the axis through the mass center of the tip body, perpendicular to x_1–x_2 plane
$\bar{b}_{s-1} = b_{s-1}/l$ normalised distance between middle lines of 1st and sth beams
$u'_s(\bar{x}_1,\bar{t}) = \partial u_s(\bar{x}_1,\bar{t})/\partial \bar{x}_1$ normalised extensional strain of sth beam
$v'_s(\bar{x}_1,\bar{t}) = \partial v_s(\bar{x}_1,\bar{t})/\partial \bar{x}_1$ normalised lateral deflection angle of sth beam in the x_2 direction, derivative of $v_s(\bar{x}_1,\bar{t})$ with respect to $\bar{x}_1$
$\dot{u}_s(\bar{x}_1,\bar{t}) = \partial u_s(\bar{x}_1,\bar{t})/\partial \bar{t}$ normalised derivative of $u_s(\bar{x}_1,\bar{t})$ with respect to $\bar{t}$
$\dot{v}_s(\bar{x}_1,\bar{t}) = \partial v_s(\bar{x}_1,\bar{t})/\partial \bar{t}$ normalised derivative of $v_s(\bar{x}_1,\bar{t})$ with respect to $\bar{t}$
$\bar{\omega} = \sigma\omega$ normalised radian natural frequency

beam-column frame with many beams and a tip body. For this purpose, the vibration behaviour of the beam-column frame, which is composed of n beams joined by a tip body, is mathematically investigated. Through this mathematical deployment, a combination of homogeneous and linear algebraic equations which can yield modal frequency and vibration mode is obtained. As a result, when whole beams are identical and symmetrically located at the upper and lower parts of the mass center of a tip body, two characteristic equations, associated with pure axial (PA) vibrations (all beams are equally and simultaneously compressed and expanded) and coupled axial-bending (CAB) vibrations (all beams are equally and simultaneously deflected with axial deformations), are obtained. On conditions of non-identical beams and non-symmetrical beam placement, a characteristic equation which explains coupled axial-bending (CAB) vibrations (all beams are deflected with axial deformations) is obtained.

The result of such an analytical approach is compared with a simulation by a finite element method (FEM): it is demonstrated that this study expresses the vibration characteristics of this model well. Finally, it is shown that this study is effectively applied to the analysis of vibration characteristics for the beam configuration of optical pickup actuators.