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Torsional and longitudinal frequency and wave response of microtubules based on the nonlocal continuum and nonlocal discrete models

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A B S T R A C T

In this paper, the size-effects in the torsional and axial response of microtubules by using the nonlocal continuum rod model is investigated. To this end, continuous and discrete rod models are performed for modeling of microtubules. A simple finite element procedure is used for modeling and solution of nonlocal discrete system equation for microtubules. The influence of the small length scale on the vibration frequencies is examined both torsional and axial vibration cases. Some parametric results are also presented for examination of the accuracy and performances of discrete and continuous models.

1. Introduction

It is well known that microtubules (MTs), microfilaments and intermediate filaments are the main components of cytoskeleton [1]. Microtubules are proteins organized in a network that is interconnected with microfilaments and intermediate filaments to form the cytoskeleton structures [2,3]. The mechanical properties of microtubules play an important role in process such as cell division and intracellular transport [4–7].

Microtubules are the most rigid of the cytoskeleton filaments and have the most complex structure [8]. There have been a number of experimental and mathematical studies in recent past 10 years dealing with the mechanical properties of MTs [9–15]. The structure of microtubules is cylindrical and it typically involves 13 parallel protofilaments which are connected laterally into hollow tubes as shown in Fig. 1. MTs are considered as hollow cylinders having 25 nm external and 15 nm internal diameters [13–17]. The length of MTs can vary from tens of nanometers to hundreds of microns. Furthermore, MTs are considered as self-assembling biological nanotubes that are essential for cell motility, building the cytoskeleton, cell division and intracellular transport. The average Young's modulus of a microtubules is $\sim$2.0 GPa [15–19]. Among the three types of cytoskeleton filaments, microtubules are the most rigid. The bending rigidity of microtubules is about 100 times that of intermediate and actin filaments. As parallel to rapid developments in nanotechnology and computational methods, some mechanical models have applied for analysis of MTs [20–27]. Present author also solved bending and buckling problems of protein microtubules based on higher-order continuum theory [28–31].

It is agreed that the classical elasticity theory is not suitable for modeling of such type nano-sized structures mentioned above. In the classical elasticity theory, only macroscopic effects are taken into consideration. It is shown that by some experimental and theoretical studies, length scale parameters play a major role in mechanical behavior of microstructure

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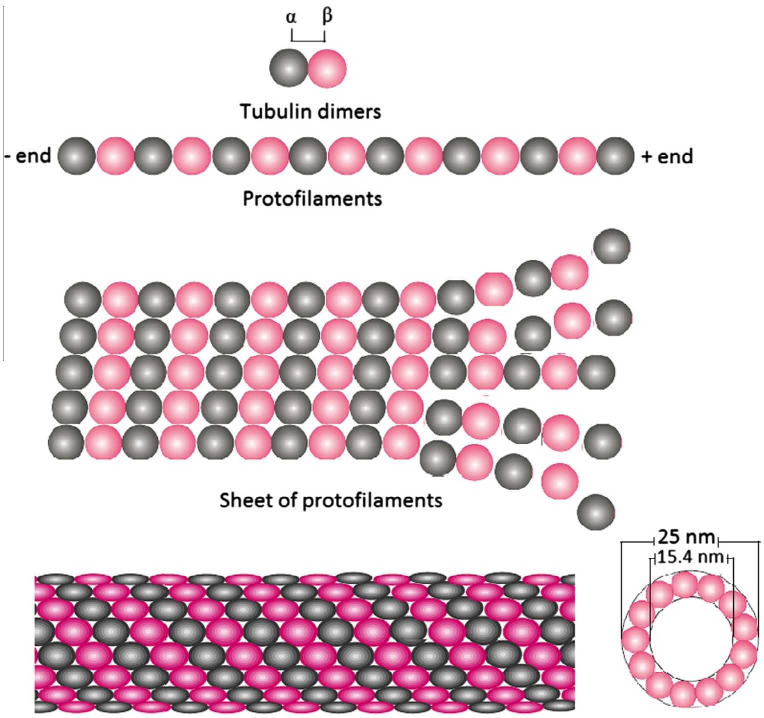

Fig. 1. A typical protein microtubules.

[32–35]. For example, when the characteristic wavelength of loading is comparable to the micro-scaled length of material, size effect will be more important in the mechanical modeling of such structures [35,36].

It is known that classical elasticity theory does not take into consideration the internal length scale effect of microstructure [37–40]. In order to introduce the size effect to the governing equations, material length scale parameters must be taken into account. Atomistic and molecular dynamic simulation models or hybrid atomistic–continuum models are computationally expensive [41–48]. Furthermore, controlled experimental studies are very difficult task for nano-scale devices in many conditions.

In recent years, higher-order continuum theories have been widely used for a wide range of engineering applications, such as modeling of micro or nano scaled structures, screw dislocations, modeling of carbon nanotubes, analysis of MEMS and NEMS, vibration control for micro devices, analysis of atomic force microscope, and ultra thin films.

In the present study, torsional and axial free vibration analyses of microtubules are investigated. The governing equations of motion based on the nonlocal elasticity theory are obtained both the continuous and discrete modeling. Solution of continuous model is obtained using the analytical method. However, simple finite element procedure is applied for nonlocal discrete equation of motion for microtubules. To the best knowledge of authors, it is the first time in the literature; the present analytical and finite element study for torsional and axial free vibration of MTs based on the nonlocal elasticity theory.

2. Nonlocal elasticity theory

It is known that the stress state of any body at a point x is related to strain state at the same point x in the classical elasticity. Namely, the constitutive equations of classical (macroscopic) elasticity are an algebraic relationship between stress and strain components. However, this theory is not conflict the atomic theory of lattice dynamics and experimental observation of phonon dispersion. As stated by Eringen [32], the linear theory of nonlocal elasticity leads to a set of integro-partial differential equations for the displacement fields of homogeneous, isotropic bodies. The solutions of these equations are difficult, in general. However, these equations can be reduced to a set of singular partial equations for some type of kernels. Thus, these spatial integrals in constitutive equations of nonlocal elasticity theory can be transformed to equivalent differential constitutive equations under certain conditions.

According to the nonlocal elasticity theory of Eringen, the stress at any reference point in the body depends not only on the strains at this point but also on strains at all points of the body. This definition of the Eringen's nonlocal elasticity is based on the atomic theory of lattice dynamics, and some experimental observations on phonon dispersion. In this theory, the long range force about atoms is considered, and thus internal scale effect is introduced in the constitutive equation. In this theory, the fundamental equations involve spatial integrals which represent the weighted averages of contributions of related strain