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A penalty-free method with line search for nonlinear equality constrained optimization

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ABSTRACT

A new line search method is introduced for solving nonlinear equality constrained optimization problems. It does not use any penalty function or a filter. At each iteration, the trial step is determined such that either the value of the objective function or the measure of the constraint violation is sufficiently reduced. Under usual assumptions, it is shown that every limit point of the sequence of iterates generated by the algorithm is feasible, and there exists at least one limit point that is a stationary point for the problem. A simple modification of the algorithm by introducing second order correction steps is presented. It is shown that the modified method does not suffer from the Maratos' effect, so that it converges superlinearly. The preliminary numerical results are reported.

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1. Introduction

In many optimization problems, the variables are interrelated by physical laws like the conservation of mass or energy, Kirchhoffs voltage and current laws, and other system equalities or inequalities that must be satisfied, see [1,2,11–13,17,22,25,29]. In this paper, we consider the following nonlinear programming problem with general nonlinear equality constraints

$$\begin{aligned} &\min f(x), \\ &s.t.\ c(x) = 0, \end{aligned} \tag{1.1}$$

where $x \in R^n, f : R^n \to R, c : R^n \to R^m$ are twice continuously differentiable.

Many efficient penalty function methods exist for solving problem (1.1). For example, sequential unconstrained optimization methods based on various penalty functions, and sequential quadratic programming (SQP) methods that use either line search or trust-region strategies [24]. The effectiveness of these so-called penalty-type methods hinges on how well the initial penalty parameter is chosen and how "intelligently" it is updated during the course of minimization. To avoid the selection of the penalty parameter, some authors research the technique without the penalty function, for example, see [4,7,8,14–16,20,26–28,30,31]. The methods which do not use any penalty function are called the penalty-free-type ones. Filter methods are important category of the penalty-free-type methods, for example, see [4,7,14–16,21,28,30,31].

Filter methods, which were introduced by Fletcher and Leyffer [15] in 1997, have been well studied and proven to be successful for solving constrained optimization problems. For example, Gould et al. [18] used filter methods to solve nonlinear equations and nonlinear least squares problems. Filter techniques have also been used to solve unconstrained problems [19], which in contrast to trust-region methods generates nonmonotone iterates with respect to the value of the objective func-

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tion, and have produced good numerical results. Chen [4,5], Gould [20], Ulbrich [27] etc., presented some other penalty-free-type methods without a filter and proved the global convergence.

Penalty-free-type methods, which do not need any penalty function, have become one of hot spots in the nonlinear optimizations. The underlying idea of this class methods is that there are two goals to determine whether a trial point is accepted or not. One is improving the feasibility and the other is reducing the value of objective function. In order to improve these two goals proportionally, we must build a relation between reducing the value of objective function and improving the feasibility.

The paper presented here gives a new method without a penalty function or a filter for the solution of (1.1), which belongs to the class of line search Newton–Lagrange method for constrained optimization. The algorithm generates new iterate points by solving linear equations and line search procedures. The new method is motivated by Wächter et al. [30,31]. Wächter et al. presented line search filter methods for nonlinear programming and analyzed the global and local convergence of their method. The main contribution of this paper is that the new method presented uses also line search but does not use any penalty function or a filter. Thus the new method does not need to reserve the filter set at every iteration. The trust-region frame is adopted in Ref. [5] and the acceptable criteria of the two algorithms are different. Under mild conditions, we analyze the global and local convergence of the algorithm presented.

Chin [8], Fletcher [15], Wächter [31] etc., have discussed that the filter technique like l_1 penalty function methods could suffer from Maratos effect when iterates are near to local solutions. As a remedy, Fletcher and Leyffer propose to improve the search direction, if the full step is rejected, by means of a second order correction which aims to further reduce infeasibility. In order to prevent the Maratos effect, we also employ second order correction steps but do not use a penalty function or a filter in this article. Under mild conditions, we analyze the local convergence of the algorithm with second order correction. The preliminary numerical results shows that the algorithm is robust and efficient.

This paper is organized as follows. In Section 2, the formal algorithm is described. In Section 3, we prove that, under mild conditions, every limit point of the sequence of iterates generated by the algorithm is feasible, and there exists at least one limit point which is a stationary point for the problem. In Section 4, we study the local superlinear convergence of the algorithm with second order correction. Some numerical results for problems from [6] are reported in Section 5.

Throughout the paper, $\|.\|$ denotes the Euclidean norm $\|.\|_2$. For simplicity, we also use subscripts to denote functions evaluated at iterates, for example, $f_k = f(x_k), c_k = c(x_k)$. Moreover, we denote

$$g(x) = \nabla f(x) \in R^n, \quad A(x) = (\nabla c_1(x), \nabla c_2(x), \ldots, \nabla c_m(x)) \in R^{n\times m}.$$

2. The algorithm

The Karush–Kuhn–Tucker (KKT) conditions for the problem (1.1) are given by

$$\begin{aligned} &g(x) + A(x)\lambda = 0,\\ &c(x) = 0, \end{aligned} \tag{2.1}$$

with the Lagrangian multipliers λ. Under linear independence of the constraint gradient $\nabla c(x)$, these are the first order optimality conditions for (1.1).

Given a starting point x_0, the proposed line search algorithm generates a sequence of improved estimates x_k of the solution for the problem (1.1). Therefore, at the current iterate point x_k, a search direction d_k is computed from the linearization of the KKT conditions (2.1),

$$\begin{bmatrix} H_k & A_k \\ A_k^T & 0 \end{bmatrix}\begin{bmatrix} d_k \\ \lambda_k^+ \end{bmatrix} = -\begin{bmatrix} g_k \\ c_k \end{bmatrix}. \tag{2.2}$$

Here, the symmetric matrix H_k denotes the Hessian matrix $\nabla_{xx}L(x_k, \lambda_k)$ of the Lagrangian function

$$L(x,\lambda) = f(x) + \lambda^T c(x)$$

of the problem (1.1), or an approximation to this Hessian. After a search direction d_k has been computed, a step size $\alpha_k \in (0,1]$ is determined by line search method, and then we can obtain the next iterate $x_{k+1} = x_k + \alpha_k d_k$.

Now we decompose the step d_k into orthogonal components

$$d_k = p_k + q_k,$$

where,

$$q_k = Y_k\bar{q}_k, \quad p_k = Z_k\bar{p}_k, \tag{2.3}$$

Y_k, Z_k are obtained from a QR-factorization of the matrix A_k, i.e.,

$$A_k = [Y_k\ Z_k]\begin{bmatrix} R_k \\ 0 \end{bmatrix},$$

here, $[Y_k\ Z_k] \in R^{n\times n}$ is an orthogonal matrix, $R_k \in R^{m\times m}$ is an upper triangular matrix. If the matrix A_k has full rank, then we follows from (2.2) that

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