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An axisymmetric problem of an embedded mixed-mode crack in a functionally graded magnetoelectroelastic infinite medium

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ABSTRACT

This paper investigates the problem of an axisymmetric penny shaped crack embedded in an infinite functionally graded magneto electro elastic medium. The loading consists of magnetoelectromechanical loads applied on the crack surfaces assumed to be magneto electrically impermeable. The material's gradient is parallel to the axisymmetric direction and is perpendicular to the crack plane. An anisotropic constitutive law is adopted to model the material behavior. The governing equations are converted analytically using Hankel transform into coupled singular integral equations, which are solved numerically to yield the crack tip stress, electric displacement and magnetic induction intensity factors. A similar problem but with a different crack morphology, that is a plane crack embedded in an infinite functionally graded magneto electro elastic medium, was considered by the authors in a previous work (Rekik et al., 2012) [25]. While the overall solution schemes look similar, the axisymmetric problem resulted in more mathematical complexities and let to different conclusions with respect to the influence of coupling between elastic, electric and magnetic effects. The main focus of this paper is to study the effect of material nonhomogeneity on the fields' intensity factors to understand further the behavior of graded magnetoelectroelastic materials containing penny shaped cracks and to inspect the effect of varying the crack geometry.

1. Introduction

Due to their ability to store and convert magnetic, electric and mechanical energies, magnetoelectroelastic composites combining piezoelectric and piezomagnetic phases have drawn significant attention in recent years. These smart structures are finding increasing use in several transducing, sensing and actuating applications [1]. Such magnetoelectromechanical coupling behavior may be found in some single phase materials; however, the main advantage of these composites is the resulting high electromagnetic coupling effect which may be even a hundred times larger compared to the single phase materials [2]. In general, magnetoelectroelastic composites are brittle and as a result cracks inevitably form during the manufacturing process and subsequent handling. Therefore, fracture mechanics of this class of materials has become an emerging research front.

Significant contributions in this area have been made by several authors. Zhou et al. [3] studied mode I problem of an embedded crack in an infinite piezoelectromagnetic medium assuming magneto electrically permeable crack surfaces.

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The problem of an opening crack in a piezoelectromagnetic solid was examined by Zhong and Li [4] who accounted for the crack opening effect on the electromagnetic boundary conditions resulting in a nonlinear system. Zhong [5] considered the problem of a dielectric crack embedded in a magnetoelectroelastic layer subjected to normal tension and took into account the crack opening effect on the electromagnetic crack surfaces boundary conditions. In [6] Hu et al. considered the antiplane crack problem in a magnetoelectroelastic layer sandwiched between dissimilar half spaces assuming permeable crack surfaces. The authors treated in [7] the problem of a Griffith crack embedded in the center of a magnetoelectroelastic strip subjected to longitudinal shear loading and in plane magneto electrical loading and assuming permeable and impermeable crack surfaces. Zhong and Li [8] examined the problem of an infinite magnetoelectroelastic solid with a penny shaped crack by taking into account the crack opening effect on the magneto electrical boundary conditions. Zhou et al. [9] studied the effect of two parallel interface cracks in magnetoelectroelastic material subjected to an anti-plane shear stress loading. The behavior of magnetoelectroelastic materials with multiple cracks under the coupled pyromagnetoelectromechanical loading was investigated by Wang and Han [10].

The concept of functionally graded materials (FGMs) has been proposed for coating design as an alternative to the conventional homogeneous coatings [11]. FGMs are nonhomogeneous composites manufactured in such a manner that the volume fractions of the constituent materials vary continuously along a spatial direction resulting in a smooth variation of the mechanical properties. Potential applications of FGMs include wear coating and thermal shielding applications such as gears, cams, cutting tools, high temperature chambers, furnace liners, turbines and space structures. The concept of FGMs was extended to the magnetoelectroelastic materials to improve the reliability of magnetoelectroelastic structures resulting in the development of Functionally Graded Magneto Electro Elastic Materials (FGMEEMs).

A number of authors considered FGMEEM fracture problems. The mode III crack problem in a functionally graded magnetoelectroelastic strip assuming ideal crack surface magneto electric permeability was considered by Ma et al. [12]. The same main author in [13] studied the surface crack problem in a functionally graded magnetoelectroelastic coating bonded to a homogeneous elastic substrate subjected to anti-plane mechanical and in plane magneto electrical loading and assuming ideal crack surface magneto electric permeability. Zhou and Wang [14] considered the problem of two parallel symmetrical permeable cracks in functionally graded materials under anti-plane shear loading. Feng and Su [15] examined the dynamic behavior of magneto electrically impermeable cracks in functionally graded magnetoelectroelastic plates. In another study, Feng and Su [16] studied the dynamic problem of a crack embedded in a graded magnetoelectroelastic strip assuming ideal crack surface permeability. Scattering of a harmonic anti-plane shear stress waves by a crack in functionally graded magnetoelectroelastic materials assuming purely permeable crack surfaces was investigated by Jun [17]. Zhou and Chen [18] studied mode I crack problem in FGMEEM infinite medium assuming air permeability within the crack. The anti-plane problem of a permeable crack intersecting the interface between two FGMEEM layers was examined by Li and Lee [19]. The same main author investigated in [20] the anti-plane interfacial fracture problem of a symmetrically bonded smart structure with linearly non-homogeneous magnetoelectroelastic properties. The anti-plane problem of a crack in a bonded FGMEEM strip sandwiched between two functionally graded strips assuming ideal magnetoelectrical permeability was considered by Guo et al. [21].

Because of mathematical complexity, relatively few researchers considered the mixed-mode axisymmetric crack problems in nonhomogeneous materials. Ozturk [22] considered the axisymmetric anti-plane shear crack problem in bonded nonhomogeneous material. Rekik et al. [23,24] studied the axisymmetric problems of an embedded crack in functionally graded layer bounded to a homogeneous substrate subjected to mechanical and thermal loads, respectively.

Recently, the authors in [25] studied the influence of a mixed mode plane crack embedded in an infinite medium made of a Functionally Graded Magneto Electro Elastic Material. The crack surfaces are assumed to be magneto electrically impermeable and are subjected to magneto electro mechanical loading. The mixed boundary value problem was reduced to a set of coupled integral equations using standard Fourier transform.

Since crack geometries may be straight, axisymmetric or telephone cord shaped, this paper attempts to investigate the effect of an axisymmetric penny shaped crack embedded in a functionally graded magneto electro elastic medium. As in [25], the crack surfaces are assumed to be magneto electrically impermeable and are subjected to magneto electro mechanical loading. The considered mixed-boundary value problem is solved using Hankel transform resulting in mathematical complexities different than [25] and which are expected to be useful for future studies. Results of both problems are compared in order to inspect the effect of different crack morphologies on the medium behavior. To the best of authors' knowledge, the combination of axisymmetric mixed-mode crack, functionally graded magnetoelectroelastic material and three-way coupling behavior has not been solved in the published literature to-date.

This paper is organized as follows. The formulation and the solution of the problem are described, respectively, in Sections 2 and 3. The numerical solution of the resulting singular integral equations is summarized in Section 4. The numerical results are then discussed in Section 5. Finally, concluding remarks are given in Section 6.

2. Problem description and governing equations

As shown in Fig. 1, the problem under consideration consists of an infinite graded magnetoelectro elastic medium. The problem is described in the (r,z) coordinate system. The graded medium contains an embedded crack of radius a. The material gradient is oriented along the z-direction. In addition, the magnetoelectro mechanical properties' tensors varying in the depth direction are assumed to vary with the same rate as follows: