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Non-linear oscillations of a thin-walled composite beam with shear deformation

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ABSTRACT

A geometrically non-linear theory is used to study the dynamic behavior of a thin-walled composite beam. The model is based on a small strain and large rotation and displacements theory, which is formulated through the adoption of a higher-order displacement field and takes into account shear flexibility (bending and warping shear). In the analysis of a weakly nonlinear continuous system, the Ritz's method is employed to express the problem in terms of generalized coordinates. Then, perturbation method of multiple scales is applied to the reduced system in order to obtain the equations of amplitude and modulation. In this paper, the non-linear 3D oscillations of a simply-supported beam are examined, considering a cross-section having one symmetry axis. Composite is assumed to be made of symmetric balanced laminates and especially orthotropic laminates. The model, which contains both quadratic and cubic non-linearities, is assumed to be in internal resonance condition. Steady-state solution and their stability are investigated by means of the eigenvalues of the Jacobian matrix. The equilibrium solution is governed by the modal coupling and experience a complex behavior composed by saddle noddle, Hopf and double period bifurcations.

1. Introduction

Thin-walled beam structures made of advanced anisotropic composite materials are increasingly found in the design of the aircraft wing, helicopter blade, axles of vehicles and so on, due to their outstanding engineering properties, such as high strength/stiffness to weight ratios and favorable fatigue characteristics. The interesting possibilities provided by fiber reinforced composite materials can be used to enhance the response characteristics of such structures that operate in complex environmental conditions. We consider the nonlinear response of a simply supported beam to a primary resonant excitation of its first mode. The analysis accounts for a lateral load, modal damping and two fiber orientations. The second and third natural frequencies are approximately two and three times the first natural frequency, respectively. The flexural–torsional coupling produces a quadratic and cubic nonlinearity in the governing nonlinear partial-differential equation. Because of the quadratic and cubic nonlinearity and the two-to-one and three-to-one ratio of the second and third with the first natural frequencies, the beam exhibits an internal (autoparametric) resonance that couples the first, second and third modes, resulting in energy exchange between them.

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For a comprehensive review of nonlinear modal interactions, we refer the reader to Refs. [1–3]. In this paper, we present a brief review of some of the studies of the response of systems exhibiting two-to-one and three-to-one internal resonances to primary resonant excitations.

Crespo da Silva and Glynn [4,5] developed a non-linear shear-undeformable beam model with a compact cross-section and derived a set of integro-partial-differential equations governing flexural–flexural–torsional motions of inextensional beams, including geometric and inertia nonlinearities. They used these equations and the method of multiple scales to ascertain the importance of the geometric terms [5]; they found that they cannot be neglected for the lower modes, especially the first mode. Luongo et al. [6] and Crespo da Silva and Zaretzky [7] analyzed shear and axially undeformable beams. In the last reference the flexural–torsional free motions are studied for a cantilever beam, having close bending and torsional frequencies; although beams with non-compact cross-section are considered, the warping effects are neglected. In these articles a non-linear one-dimensional polar model of compact beam is derived, capable of studying interactions between flexural and torsional motions occurring in beam-like structures in several internal resonance conditions. The non-linear planar motions and the non-linear resonance frequencies was recently investigated by Fonseca and Ribeiro [8]. They used a p-version finite element formulated for geometrically non-linear vibrations. Lopes Alonso and Ribeiro [9] continued this last work [8] for the free vibrations of clamped–clamped circular cross section beams using hierarchic sets of displacement shape functions and that simultaneously considers bending, torsion and longitudinal deformation. In this case, they employed the harmonic balance method to show the variation of the bending and torsional shapes of vibration with the non-linear natural frequency. The effects of the warping function, longitudinal displacements of second order and shear deformations on the nonlinear bending-torsion vibrations of rectangular cross section was analyzed in the work of Stoykov and Ribeiro [10]. In relation to thin-walled beams, Di Egidio et al. [11,12] presented the dynamic response of an open cross-section beam divided in two works. They developed a shear undeformable thin-walled beam where the effects of non linear in-plane and out-of-plane warping and torsional elongation were included in the model [11]. The dynamic coupling phenomena in conditions of internal resonance was analyzed in the second part [12]. Machado and Saravia [13] investigated the effect of shear deformation on the frequency–response curves of a thin-walled composite beam. They showed that the equilibrium solutions are influenced by the transverse shear effect. The amplitude of vibration is reduced significantly when this effect is ignored, altering the dynamic response of the beam.

In this paper, a geometrically non-linear beam model is used to study three dimensional large amplitude oscillations. It is shown that the system exhibits periodic and quasiperiodic responses for a typical range of parameter values. The limit cycles which born from the Hopf bifurcation are analyzed. A schematic bifurcation diagrams for the orbits of the modulation equations is presented. The mathematical model is valid for symmetric balanced laminates and incorporates, in a full form, the effects of shear flexibility. In order to perform the nonlinear dynamic analysis the Galerkin procedure is used to obtain a discrete form of the equations of motion. Multiple time scales method is used to obtain modulation-phase equations [14] and the reconstitution method proposed in [15] is adopted to return to the true time domain. Steady state solutions and their stability are studied by using the model proposed. For principal parametric resonance of the first mode, the influence of internal resonance is illustrated in frequency and amplitude plots.

2. Kinematics

A straight thin-walled composite beam with an arbitrary cross-section is considered (Fig. 1). The points of the structural member are referred to a Cartesian co-ordinate system $(x, \bar{y}, \bar{z})$, where the x-axis is parallel to the longitudinal axis of the beam while $\bar{y}$ and $\bar{z}$ are the principal axes of the cross-section. The axes y and z are parallel to the principal ones but having their origin at the shear center (defined according to Vlasov's theory of isotropic beams). The co-ordinates corresponding to points lying on the middle line are denoted as Y and Z (or $\bar{y}$ and $\bar{z}$). In addition, a circumferential co-ordinate s and a normal co-ordinate n are introduced on the middle contour of the cross-section.

$$\bar{y}(s,n) = \overline{Y}(s) - n\frac{dZ}{ds}, \quad \bar{z}(s,n) = \overline{Z}(s) + n\frac{dY}{ds}, \tag{1}$$

$$y(s,n) = Y(s) - n\frac{dZ}{ds}, \quad z(s,n) = Z(s) + n\frac{dY}{ds}. \tag{2}$$

On the other hand, y_0 and z_0 are the centroidal co-ordinates measured with respect to the shear center.

$$\bar{y}(s,n) = y(s,n) - y_0, \quad \bar{z}(s,n) = z(s,n) - z_0. \tag{3}$$

The present structural model is based on the following assumptions [16]:

(1) The cross-section contour is rigid in its own plane.
(2) The warping distribution is assumed to be given by the Saint–Venant function for isotropic beams.
(3) Flexural rotations (about the $\bar{y}$ and $\bar{z}$ axes) are assumed to be moderate, while the twist ϕ of the cross-section can be arbitrarily large.
(4) Shell force and moment resultants corresponding to the circumferential stress σ_{ss} and the force resultant corresponding to γ_{ns} are neglected.