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# A new method for ranking fuzzy numbers and its application to group decision making

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ABSTRACT

In this paper, a new method for comparing fuzzy numbers based on a fuzzy probabilistic preference relation is introduced. The ranking order of fuzzy numbers with the weighted confidence level is derived from the pairwise comparison matrix based on 0.5-transitivity of the fuzzy probabilistic preference relation. The main difference between the proposed method and existing ones is that the comparison result between two fuzzy numbers is expressed as a fuzzy set instead of a crisp one. As such, the ranking order of $n$ fuzzy numbers provides more information on the uncertainty level of the comparison. Illustrated by comparative examples, the proposed method overcomes certain unreasonable (due to the violation of the inequality properties) and indiscriminative problems exhibited by some existing methods. More importantly, the proposed method is able to provide decision makers with the probability of making errors when a crisp ranking order is obtained. The proposed method is also able to provide a probability-based explanation for conflicts among the comparison results provided by some existing methods using a proper ranking order, which ensures that ties of alternatives can be broken.

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## 1. Introduction

The method for comparing and ranking fuzzy numbers was first studied by Jain in 1976 [1], and has proliferated since [2–14]. As Matarazzo and Munda [15] pointed out "a key issue in operationalizing fuzzy set theory is how to compare fuzzy numbers". This is evidenced by the extensive coverage of ranking methods in various areas, which include optimization [16,17], artificial intelligence [18,19], approximate reasoning [20], decision-making [21,22], and socio economic systems [23] et al. However, the existing methods exhibit certain limitations, such as inconsistency with human intuition [24], indiscrimination [25], difficulty of interpretation [26], or even unreasonable results [27,28].

Many of the existing methods strive to achieve a crisp output based on the initial comparison between fuzzy numbers by mapping a fuzzy number into a real-valued number, such as the centroid-based method [29,30], distance-based method [2,24,27,31,32], area-based method [7,33], and other real-valued index methods [10,34]. These ranking methods provide the crisp results based on the only constraint of the neutral attitude of decision makers (i.e., without considering the preferences of decision makers) during the evaluation process. A number of preference ranking methods have been suggested [3,12,35] by taking into consideration decision makers' assessment attitudes for different decision-making problems.

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However, these preference ranking methods are still based only on the preference functions derived from a simplistic linear combination of two extreme (optimistic and pessimistic) points. Other methods [11,12,25,28] to represent decision-makers' preferences more explicitly by analyzing the satisfaction function or weighting function have also been proposed. As an example, Yager [11] presented a fuzzy number ranking method using weighting functions, while Liu [12] proposed another one that integrates both the centroid method and weighting function method. Motivated by the satisfaction function, Huynh et al. [25] proposed a probability-based method for comparing fuzzy numbers in decision making problems. Subsequently, Chen and Lee [28] improved the work in [25] by proposing a new method to rank fuzzy sets, which uses fuzzy targets based on the likelihood comparison relations.

While the comparison results of these methods are more precise, they still yield crisp values despite taking into account decision makers' subjective preferences. This causes loss of information; hence leading to some unreasonable or counter-intuitive comparison results (see examples 2 and 4 for more details). In order to preserve more information within the comparison results, a soft (fuzzy) comparison result attached with a membership function indicating the confidence degree of individual decision maker is introduced in this paper. On the other hand, since fuzzy numbers represent the subjective knowledge captured in the evaluation of alternatives, it is appropriate to maintain a soft (fuzzy) comparison result. This ensures flexibility in decision-making, with the range of decision makers' preferences being preserved.

In this paper, a new approach to provide a comparison result between two fuzzy numbers with an outcome in the form of a fuzzy set is proposed. Its corresponding crisp result with a probabilistic decision error can also be obtained precisely by attaching decision makers' confidence degrees. The advantage of the proposed method lies in its flexibility whereby objective information, such as the probability of $A$ is greater than $B$, can be combined with subjective information expressed as a weighting function of decision-makers' preferences. Based on the property of 0.5-transitivity of the proposed method, we further extend the fuzzy pairwise comparison result into a graph-based ranking method for ranking $n$ fuzzy numbers.

The rest of this paper is organized as follows. In Section 2, some definitions and background information are given. A probability-based ranking method for intervals is described. The proof to show that the interval ranking method satisfies 0.5-transitivity of the fuzzy probabilistic preference relation is given. In Section 3, a new method and a comparison algorithm for fuzzy numbers based on the fuzzy probability preference relation is presented. In Section 4, a graph-based ranking method for ranking $n$ fuzzy numbers is introduced. Its corresponding path-finding algorithm and some examples for illustrating the algorithm are also given. In addition, some logical properties related to an ordering method are examined to verify the validity of the new ranking method. Conclusions and recommendations for future research are presented in Section 5.

## 2. Preliminaries

A fuzzy number is a fuzzy subset of the real line, $R$, with a convex and continuous membership function. The family of fuzzy numbers is denoted by $\mathbb{F}$.

**Definition 1.** [36] For any given $\tilde{A} \in \mathbb{F}$, the membership function of $\tilde{A}$, $\mu_{\tilde{A}}(x)$, is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} f^L_{\tilde{A}}(x), & a \leqslant x < b \\ 1, & b \leqslant x \leqslant c \\ f^R_{\tilde{A}}(x), & c < x \leqslant d \\ 0, & otherwise \end{cases}$$

where $f^L_{\tilde{A}}:\ [a,b] \to [0,1]$; is a monotonic, continuous, and strictly increasing mapping function from $R$ to the closed interval $[0,1]$; and $f^R_{\tilde{A}}:\ [c,d] \to [0,1]$ is also a monotonic, continuous, but strictly decreasing mapping function from $R$ to the closed interval $[0,1]$. The inverse functions of $f^L_{\tilde{A}}$ and $f^R_{\tilde{A}}$ are denoted by $g^L_{\tilde{A}}$ and $g^R_{\tilde{A}}$, respectively. Since $f^L_{\tilde{A}}$ and $f^R_{\tilde{A}}$ are strictly increasing and decreasing mapping functions, respectively, their inverse functions exist, and are also monotonic.

**Definition 2.** [37] Let $\tilde{A}$ be a fuzzy set defined on its universe of discourse, $X$. For any given real number $\alpha \in (0,1)$, the $\alpha$-cut of $\tilde{A}$, denoted as $A_\alpha$, is defined by:

$$A_\alpha = \{x | \mu_A(x) \geqslant \alpha\} \quad \forall x \in X$$

The $\alpha$-cut of $\tilde{A}$ is a crisp set, denoted as an interval $[A_\alpha, \bar{A}_\alpha]$, where $A_\alpha = \inf\{x|\mu_{\tilde{A}}(x) \geqslant \alpha\}$ and $\bar{A}_\alpha = \sup\{x|\mu_{\tilde{A}}(x) \geqslant \alpha\}$. Obviously, $[A_\alpha, \bar{A}_\alpha] \subseteq R$ is an interval on the real line. When $\alpha = 0$, the support of $\tilde{A}$ is defined as [38]: supp $(\tilde{A}) = \overline{\{x|\mu_{\tilde{A}}(x) > 0\}}$, where $\overline{\{x|\mu_{\tilde{A}}(x) > 0\}}$ is the closure of set $\{x|\mu_{\tilde{A}}(x) > 0\}$. When $\alpha = 1$, the kernel of $\tilde{A}$ is defined as $ker(\tilde{A}) = \{x|\mu_{\tilde{A}}(x) = 1\}$.

By using the decomposition theorem proposed by Zadeh [39], a fuzzy set, $\tilde{A}$, can be represented as

$$\tilde{A} = \bigcup_{\alpha \in [0,1]} \alpha A_\alpha, \text{ where } \alpha A_\alpha(x) = \begin{cases} \alpha & if\ x \in A_\alpha \\ 0 & if\ x \notin A_\alpha \end{cases} \tag{1}$$

Based on this theorem, if all the $\alpha$-cuts of a fuzzy set can be determined, then the fuzzy set itself can be specified. Therefore, determining a fuzzy set is equivalent to determining all its $\alpha$-cuts for any $\alpha \in [0,1]$.

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