Applied Mathematical Modelling 38 (2014) 4559–4573

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Capability analysis of the variable measurement system with fuzzy data

Hadi Moheb-Alizadeh *

Department of Industrial and Systems Engineering, North Carolina State University, Raleigh, NC 27695, USA

ARTICLE INFO

Article history:
Received 10 November 2012
Received in revised form 21 December 2013
Accepted 11 March 2014
Available online 25 March 2014

Keywords:
Measurement system analysis (MSA)
Extension principle
Fuzzy number
Ranking method
Non-linear programming

ABSTRACT

The aim of this paper is to propose an approach to analyze capability of the variable measurement system in fuzzy environment, where the data acquired from the measurement process under study are assumed fuzzy numbers. To accomplish this goal, a pair of non-linear programming problems is formulated based on Zadeh's extension principle to compute α-level cuts of assessment criteria, which are frequently used to analyze capability of the variable measurement system in practice. The membership functions of these criteria are then constructed analytically by numerating different values of α. The capability assessment criteria discussed in this paper include repeatability, reproducibly, *GRR%* and C_{gk}. In the next step, a method for ranking fuzzy numbers is exploited to evaluate whether capability of the variable measurement system is satisfactory in fuzzy environment or not. Since fuzzy measures are gathered from the measurement system in a more realistic situation in which all variations and unexpected conditions are taken into account, it is shown using an empirical example that incorporating fuzziness into measurement data results in a more accurate capability analysis.

1. Introduction

All decisions made about the status of a manufacturing process, including accepting or rejecting production parts, initial setting up or re-setting up production equipments, inspecting samples and so forth, are based on data drawn from the process. This reflects the importance of quality of obtained data. Nowadays, almost all manufacturing organizations calibrate the control instruments used to extract the required data, whereas the measurement instrument is just one component of a measurement system. The measurement system includes appraisers, production parts, measurement instruments, methods and physical environment (such as light, heat, etc.). Hence, the adequacy of just measurement instrument does not solitarily assure the correctness of a measurement system. Each of the abovementioned components may significantly influence the measurement system performance and consequently, the quality of collected data. Measurement system analysis (MSA) is a statistical approach to investigate effects of the components and their interactions on the measurement system performance.

Based on the type of its outcome, a measurement system is generally categorized into two classes including variable and attribute measurement systems. A variable measurement system which is the subject of the present study takes into account a continuum value as measurement outcome. On the other hand, one of a finite number of categories is regarded as measurement outcome in an attribute measurement system. Capability of the variable measurement systems is examined by several criteria such as repeatability, reproducibility, *GRR%* and C_{gk}.

* Tel.: +1 (919) 274 6937.
E-mail address: hmoheba@ncsu.edu

http://dx.doi.org/10.1016/j.apm.2014.03.017

Measurement systems are traditionally analyzed using precise data. However, there may be many situations in which we are not able to collect precise and certain measurement data. As JCGM 100:2008 [1] mentions, even if all of the known or suspected components of measurement errors have been evaluated and the appropriate corrections have been applied, there still remains an uncertainty about the correctness of the stated result, that is, a doubt about how well the measurement result represents the precise value of the quantity being measured. In practice, such an uncertainty may be resulted from several sources including: incomplete definition of the part being measured (measurand), imperfect realization of the measurand definition, inadequate knowledge of the effects of environmental conditions on the measurement, inexact values of measurement standards and reference materials, approximations and assumptions incorporated in the measurement method and procedure [1], special structure or shape of the measurand, inherent variability in the measurand, human judgment and subjectivity and so forth. One way to take into account such an uncertainty in analyzing capability of a measurement system is to deal with imprecise or vague data. In this case, the outcome of a measurement system can be represented as a linguistic term, such as either "a range between 270.15 and 270.3" in a variable-measured production process, or "good", "medium" and "bad" in an attribute-measured production process. This uncertain situation can be properly modeled by fuzzy set theory initially proposed by Zadeh [2]. In this case, the data drawn from the variable measurement system are represented as fuzzy numbers instead of common precise quantities.

To the best of our knowledge, no research studying the variable measurement system in fuzzy environment is observed in the literature, while other quality engineering techniques such as failure mode and effect analysis (FMEA), quality function deployment (QFD), control charts and statistical quality control (SPC) have been comprehensively addressed and investigated in fuzzy environment. For example, Xu et al. [3] presented a fuzzy logic-based method for FMEA to address interdependencies among various failure modes with uncertain and imprecise information. Liu et al. [4] proposed an FMEA using the fuzzy evidential reasoning (FER) approach and grey theory to improve attaining various opinions of FMEA team members and determining risk priorities of the failure modes. Chen and Ko [5] proposed fuzzy nonlinear programming models based on Kano's concept to determine the accomplishment levels of parts characteristics in quality function deployment (QFD) technique. Liu [6] integrated fuzzy QFD and the prototype product selection model to develop a product design and selection (PDS) approach. He adopted the α-cut operation to calculate the fuzzy set of each component and considered engineering characteristics and the factors involved in prototype product selection. In statistical quality control area, Moheb-Alizadeh and Fatemi-Ghomi [7] explored the impact of various transformation methods on power of control charts and concluded that fuzzy mode and fuzzy average transformation methods lead to the most and least powerful control charts, respectively. Shu and Wu [8] proposed the fuzzy $\bar{X}$ and R control charts, whose fuzzy control limits are obtained on the basis of the results of resolution identity. The interested reader is referred to [9–13] to investigate more quality engineering techniques in fuzzy environment.

As the sole study, this paper endeavors to analyze capability of the variable measurement system in fuzzy environment In this regard, it is supposed that the data obtained from such a measurement system are fuzzy numbers. Applying Zadeh's extension principle [14,15], a pair of non-linear mathematical programming problems is formulated to compute the lower and upper bounds of α-level cuts of assessment criteria, which are prevalently used for capability analysis of the variable measurement system in practice. These criteria include repeatability, reproducibility, $GRR\%$ and C_{gk}. Afterward, the membership functions of these criteria in fuzzy environment are derived numerically by counting different values of α. In order to evaluate whether capability of the variable measurement system is acceptable with fuzzy data or not, a method for ranking fuzzy numbers is exploited. Since use of fuzzy data in capability analysis is more realistic and practical, it is shown using an empirical example that the proposed approach provides a more accurate tool for examining capability of the variable measurement system. The developed approach can be employed in any measurement laboratory on any type of measurement device, where taking into account the measurement uncertainty is concerned. Hence, the application of the proposed approach may widely rang from pharmaceutical and biomedical industries, auto-industry and aerospace industry to nano-manufacturing. In all these industries, it is required to derive the most precise and correct measures from a measurement system to have a fair judgment on whether the product under measurement is hitting the acceptability criteria or not.

The rest of this paper is organized as follows: Section 2 gives preliminary concepts about variable measurement system. Section 3 describes how to use extension principle to derive α-level cuts of assessment criteria of capability in fuzzy environment. A ranking method for fuzzy numbers used to evaluate capability criteria with fuzzy data is given in Section 4. Then, Section 5 represents a numerical example to show how to perform the proposed approach in a practical context. Finally, some conclusions and recommendations for future works are given in Section 6.

2. Measurement systems analysis

This section presents the theory of variable measurement system analysis when crisp measurement data are applied. Measurement systems analysis (MSA) is viewed as an important component for many quality initiatives. As a part of ISO/TS 16949 and AIAG [16] standards, it is defined as an experimental and mathematical approach for determining how much the variation resulted from measurement process contribute to total process variability. Measurement systems are generally classified into two distinct groups including attribute measurement system and variable measurement system, on which the present paper concentrates. Similar to all other processes, the measurement systems are influenced by both random and systematic sources of variation. The acronym S.W.I.P.E is used to represent the six potential sources of variation in measurement