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Availability optimization of a redundant system through dependency modeling ☆

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ABSTRACT

This paper presents a constructive approach to optimize the availability of a system through modeling the dependency of the components. Our goal is to minimize the system cost under the constraint that system availability must not be less than a given level. In particular, the components are dependent of each other. A function noted as the dependence function is introduced to model the dependency. It is demonstrated that, for a general form of the system cost, the dependence function guarantees a finite set of feasible solutions. An approach is then developed with the help of the dependence function to obtain the optimal solution. The resolution is illustrated by an interesting example, in which the system cost depends on the strength of the dependency. Our study reveals that the dependency is an essential and effective option to improve system reliability. Moreover, the modeling of dependency, i.e. the introduction of the dependence function is valuable for resolving the optimization problem.

1. Introduction

Modern systems function with uncertainty. Due to the importance of their functions, a high level of system reliability/availability (SRA) is often required. In practice, there are many options to ensure the reliable functioning of a system, e.g. the enhancement of the reliability of the component, the provision of redundant units, and the maintenance operations. These options, however, largely increase the system cost. It is therefore necessary to tradeoff between the cost and the level of SRA.

The study of SRA optimization has been a natural intersection of Operations Research and Reliability Engineering. Given the structure of a system, the optimization experts formulate SRA of the structure and then focus on resolving the optimization problem, as shown in the reviews [1,2]. On the other hand, the system engineers adopt dynamic options to improve SRA upon the analysis of the system evolution [3].

It is somehow surprising that the system has been interpreted differently by the two communities. The system is modeled as a static structure by optimization experts, whereas as an evolution process by system analysts. This bifurcation arises

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from the view of the dependency [4]. If the evolution of a system depends on temporal factors, a dynamic model has to be adopted. Otherwise, the dynamic model can recur to a static one.

Note that the dependency can be observed in a wide range of engineering practices [5–10]. For example, the failure/adding of a redundant component may alter SRA both by the loss/gain of the component's reliability and by the redistribution of system loading. A survey of dependency models concerning SRA can be found in [11]. Indeed, the dependency invites us to investigate more accurately the behavior of the system. For SRA optimization, the dependency is also significant as it can improve the system reliability in an economic way [12]. Hence, the dependency becomes attractive for both system modeling and SRA optimization.

To evaluate the dependency for SRA optimization, this paper considers a maintainable system composed of redundant and dependent components. Our goal is to minimize the system cost, subject to the requirement on system availability. The redundant dependency, defined as the dependency of redundant components [12], is described by a function, namely the dependence function, where the function parameter stands for the dependency strength.

The paper is organized as follows. The system and the dependency are modeled in Section 2. The availability of the system is analyzed in Section 3 with emphasis on its quantitative properties. In Section 4, the optimization problem is formulated and the resolution procedure is progressively developed. An example is then presented and the computation results are discussed in Section 5. The conclusion and the perspectives are finally addressed in Section 6.

2. System description and dependency modeling

2.1. System description

A system is composed of n identical redundant components. The failure distribution of the component is exponential with the nominal hazard rate λ. There is one repairperson to fix up the failed component. The repairperson can handle only one component at one time, where the repair time is exponentially distributed with the repair rate μ. Thus, the system can be modeled by Continuous-Time Markov Chain (CTMC) [13]. Let λ_{n-i} be the hazard rate of the component at the system state $(n-i)$. The state transition diagram of the system can be shown as Fig. 1:

Let $\boldsymbol{\pi}_S$ be the distribution of the stationary state of the system. The system equations will be:

$$\begin{cases} \mathbf{D} \bullet \boldsymbol{\pi}_S = \mathbf{0} \\ \sum_{i=0}^{n} \pi_S^i \equiv 1 \end{cases}, \quad \mathbf{D} = \begin{bmatrix} -n\lambda_n & \mu & & & \\ n\lambda_n & -(n-1)\lambda_{n-1} - \mu & \mu & & \\ & \cdots & \cdots & \cdots & \\ & & 2\lambda_2 & -\lambda_1 - \mu & \mu \\ & & & \lambda_1 & -\mu \end{bmatrix}, \tag{1}$$

where $\mathbf{D}$ is the transition matrix of the system.

From Eq. (1), the state distribution of the system $\boldsymbol{\pi}_S$ can be obtained as:

$$\boldsymbol{\pi}_S = \frac{1}{S_n}\begin{bmatrix} 1 & \frac{n\lambda_n}{\mu} & \frac{n(n-1)\lambda_n\lambda_{n-1}}{\mu^2} & \cdots & \frac{n(n-1)\cdots 2\cdot\lambda_n\lambda_{n-1}\ldots\lambda_2}{\mu^{n-1}} & \frac{n!\cdot\lambda_n\lambda_{n-1}\ldots\lambda_1}{\mu^n} \end{bmatrix}^T, \tag{2}$$

where $S_n = 1 + \sum_{j=1}^{n}\prod_{i=0}^{j-1}\frac{(n-i)\lambda_{n-i}}{\mu}$.

2.2. Redundant dependency

The dependency refers to the interactions between the components in a system. In this paper, a special dependency is studied, i.e. the redundant dependency, which is originally defined in [12] as:

The dependency of a system is called the redundant dependency if any of the components can be viewed as a redundancy of one another component.

The dependency influences the system evolution in an essential way. If there is only one component working in the system, the hazard rate of the component will be the nominal value λ. If there are several dependent components working in the system, the hazard rate of the components varies as the dependency functions.

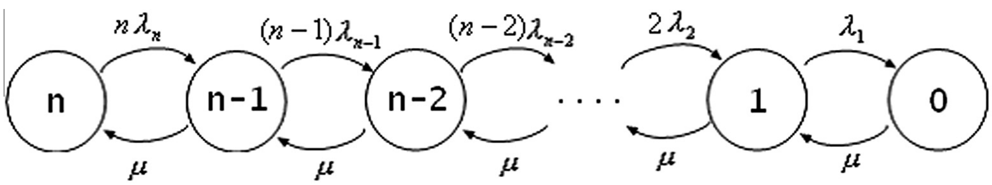

Fig. 1. State transition diagram of the system.