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Single-machine group scheduling with processing times dependent on position, starting time and allotted resource

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ABSTRACT

This article considers scheduling problems on a single machine with learning effect, deteriorating jobs and resource allocation under group technology (GT) assumption. We assume that the actual processing time of a job depends on the job position, the group position, the starting time and the amount of resource allocated to them concurrently, and the actual setup times of groups depend on the group position and the amount of resource allocated to them concurrently. Two resource allocation functions are examined for minimizing the weighted sum of makespan and total resource cost. We prove that the problems have polynomial solutions under the condition that the number of jobs in each group are the same.

1. Introduction

In traditional scheduling theory and problems, the job processing times are fixed and have constant values. However, we often encounter production systems in which job processing times may be changing due to the phenomenon of deterioration (i.e., the later a jobs starts, the longer it takes to process) and/or learning (i.e., the actual processing time of a job gets shorter, provided that the job is scheduled later) and/or resource allocation (i.e., the actual processing time of a job gets shorter, provided that the job is allocated by additional resource). Extensive surveys of different scheduling models and problems involving deteriorating jobs and/or learning can be found in Gawiejnowicz [1], Biskup [2] and Janiak et al. [3]. Extensive surveys of research related to scheduling models and problems with resource allocation (controllable processing times) were provided by Shabtay and Steiner [4] and Janiak et al. [5]. More recent papers that have considered scheduling problems with deteriorating jobs and/or learning effects and/or resource allocation (controllable processing times) include Yang [6], Wang et al. [7], Shen and Wu [8], Wang et al. [9], Edis et al. [10], Lee and Yang [11], Wang and Wang [12], Yang et al. [13], and Wang and Wang [14].

On the other hand, the production efficiency can be increased by group technology, i.e., grouping various parts and products with similar designs and or production processes. Yang and Yang [15] considered single-machine scheduling with the effects of deterioration and learning under group technology, i.e., the actual processing time of job J_{ij} when it is scheduled in

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the rth group and in the lth position within the group is given by $p_{ij}^A = p_{ij}r^{a_i}$ and $p_{ij}^A = p_{ij}(1 + \sum_{q=1}^{r-1} p_{i[q]})^{a_i}$, where p_{ij} is the original (normal) processing time of job J_{ij}, $a_i \le 0$ denote the learning factor of group G_i; And the actual setup time of group G_i is $s_i^A = \delta_i t$ where δ_i is the setup deterioration factor of group G_i. They showed that the makespan minimization problems can be solved in polynomial time and the total completion time minimization problems can also be solved in polynomial time under agreeable conditions. Yang et al. [16] considered group scheduling with learning and deterioration effects on a single-machine, i.e., the actual processing time of job J_{ij} when it is scheduled in the rth group and in the lth position within the group is given by $p_{ij}^A = (p_{ij} + \alpha t)r^{a_1} l^{a_2}$, where t is the starting time of job J_{ij}, $b_1 \leqslant 0$ and $b_2 \leqslant 0$ denote the learning indices of the group and job learning effect respectively, $\alpha \geqslant 0$ is a common deterioration rate for all the jobs; the actual setup time of group G_i is $s_i^A = s_i r^{a_1}$ where s_i is the normal setup time of group G_i. They showed that makespan minimization problem and the total completion time minimization problem are polynomially solvable under agreeable conditions. Yang [17] considered group scheduling models with the effects of deterioration and learning, i.e., $p_{ij}^A = p_{ij} + \alpha_i t$, $p_{ij}^A = p_{ij}r^{a_i}$ and $p_{ij}^A = (p_{ij} + \alpha_i t)r^{a_i}$, where $\alpha_i > 0$ $(a_i > 0)$ is the deteriorating factor of jobs in group group G_i; And the actual setup time of group G_i is $s_i^A = s_i r^a$ where s_i is the normal setup time of group G_i, and $a < 0$ is the common learning factor of groups. They showed that some results of the makespan (total completion time) minimization problems can be solved in polynomial time under some conditions. Kuo [18] considered the model: $p_{ij}^A = p_{ij}(1 + \sum_{q=1}^{r-1} p_{i[q]})^{a_i}$, $s_i^A = s_i r^b$ and $s_i^A = s_i r^{b_i}$ where $a_i \le 0$ denote the learning factor of group G_i, $b \le 0$ is the group-independent learning index of setup time, and $b_i \le 0$ is the group dependent learning index of setup time for group G_i. They provided polynomial time algorithms to solve the makespan minimization problems. They also provided two polynomial time algorithms to solve the total completion time minimization problems under certain conditions. Wang et al. [19] and Xu et al. [20] considered the single machine group scheduling problems with ready times, in which the processing time of a job is a function of its starting time, and the setup time of a group is assumed to be known and fixed.

Zhu et al. [21] considered single-machine group scheduling with resource allocation and learning effect, i.e., $p_{ij}^A = p_{ij}r^{b_1} l^{b_2} - \beta_{ij}u_{ij}$, $p_{ij}^A = \left(\frac{p_{ij}r^{b_1} l^{b_2}}{u_{ij}}\right)^k$, $u_{ij} > 0$, and $s_i^A = s_i r^{b_1}$, where $b_1 \leqslant 0$ and $b_2 \leqslant 0$ denote the learning indices of the group and job learning effect respectively, and u_{ij} is the amount of a non-renewable resource allocated to job J_{ij}, with $0 \leqslant u_{ij} \leqslant \bar{u}_{ij} < \frac{p_{ij}m^{b_1}(n_m)^{b_2}}{\beta_{ij}}$, where $\bar{u}_{ij}$ denote the maximum amount of resource allocated to job J_{ij} and β_{ij} is the positive compression rate of job J_{ij}, $k > 0$ is a given constant. They showed that the problems for minimizing the weighted sum of makespan and total resource cost remain polynomially solvable. They also proved that the problems for minimizing the weighted sum of total completion time and total resource cost have polynomial solutions under certain conditions. Wang et al. [22] considered group scheduling with resource allocation and deteriorating jobs on a single machine, i.e., $p_{ij}^A = p_{ij} + \alpha t - \beta_{ij}u_{ij}$, $p_{ij}^A = \left(\frac{p_{ij}}{u_{ij}}\right)^k + \alpha t$, $u_{ij} > 0$, $s_i^A = s_i + \beta t - \beta_i u_i$, $s_i^A = \left(\frac{s_i}{u_i}\right)^k + \beta t$, where $\alpha \geqslant 0$ denote the common deterioration rate for all the jobs, $\beta \geqslant 0$ denote the common deterioration rate for all the groups, $0 \leqslant u_{ij} \leqslant \bar{u}_{ij} < \frac{p_{ij}}{\beta_{ij}}$. They showed that two problems for minimizing the weighted sum of makespan and total resource cost remain polynomially solvable for a special condition. This paper extends the results of Zhu et al. [21] and Wang et al. [22], by considering a more general deterioration, learning effect and resource allocation model that includes the one given in Zhu et al. [21] and Wang et al. [22] as a special case. The phenomena of job processing times with learning effect, deterioration effect, and resource allocation simultaneously can be found in a steel production, in the process of pre-heating ingots by gas to prepare them for hot rolling on the blooming mill. Before the ingots can be hot rolled, they have to achieve the required temperature. The pre-heating time can be shortened by the increase of the gas flow intensity, i.e., the ingot pre-heating time depends on the starting moment of the pre-heating process and the amount of gas consumed during it. On the other hand, the learning effects reflect that the workers become more skilled to operate the machines through experience accumulation; and production efficiency can be increased by group technology assumption. Hence, we aim to study the scheduling model with deterioration effect, learning effect, resource allocation and group technology at the same time. (Bachman and Janiak [23], Wang et al. [24]).

The remainder of this article is organized as follows. Section 2 formulates the problem. Sections 3 and 4 derive the properties of the optimal schedules for two variants of the problem and provide solution algorithms for them. The last section contains some conclusions and suggests some future research topics.

2. Problem formulation

The problem is developed using the following notations. Additional notations will be introduced when needed throughout the paper.

m	the number of groups $(m \geqslant 2)$
G_i	the ith group, $i = 1, 2, \ldots, m$
n_i	the number of jobs in group $G_i, i = 1, 2, \ldots, m$

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