



# Singularly perturbed homotopy analysis method

Ophir Nave<sup>a,b,\*</sup>, Shlomo Hareli<sup>c</sup>, Vladimir Gol'dshtein<sup>a</sup>

<sup>a</sup>Department of Mathematics, Ben-Gurion University of the Negev, PO Box 653, Beer-Sheva 84105, Israel

<sup>b</sup>Jerusalem College of Technology (JCT), Israel

<sup>c</sup>Hebrew University of Jerusalem, Israel

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## ABSTRACT

In this paper we combined the homotopy analysis method (HAM) and the method of integral manifold (MIM) to investigate the problem of thermal explosion in two-phases poly-disperse combustible mixtures of gas with fuel droplets. The size distribution of the fuel droplets is assumed to be continuous in the form of an exponential distribution and is found from the solution of the kinetic equation for the probability density function. The system of the polydisperse fuel spray takes into account the effects of the thermal radiation and convection. By applying the HAM and the MIM, we derived an analytical solution of the system and we compared our results with the numerical solutions.

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## 1. Introduction

Most physical and engineering phenomena are modeled by non-linear differential equations. For these phenomena, it is very difficult to achieve analytical solutions. In most cases, only approximate solutions can be expected. Although at this time we have high-performance super-computers and some high-quality symbolic software, it is often more difficult to get an analytical solution than a numerical one of a given nonlinear problem [1].

Our analysis focuses on the models of thermal explosion of polydisperse fuel spray. Such models are nonlinear and involve different time scales. Therefore, the natural way of modeling these processes is to consider them as singular perturbed systems (SPS) of ordinary differential equations. In order to investigate these models, different asymptotic methods can be applied. In this paper we combine two analytical methods: the well known geometric asymptotic method (method of integral manifolds, MIM), that has been introduced by [2–4], with homotopy analysis methods that have been introduced by Liao [5]. The perturbation method, a numerical method used for solving both linear and non-linear problems, is based on assuming a small parameter. In many cases, nonlinear problems, especially physical models that are nonlinear, have no small parameter, and the solutions that obtained by the perturbation methods are valid only for small values of the small parameter. In order to overcome these difficulties, He [6] introduced a modification of the homotopy method called the homotopy perturbation method (HPM). The HPM does not depend upon a small parameter in the equations. This technique is constructed with an embedding parameter  $p \in [0, 1]$ , which is considered as a small parameter (artificial parameter). When  $p$  is increased from 0 to 1, the model of equations goes through a sequence of deformation, the solution of each which is close

\* Corresponding author at: Department of Mathematics, Ben-Gurion University of the Negev, PO Box 653, Beer-Sheva 84105, Israel. Tel.: +972 5 042 04491; fax: +972 8 647 7648.

E-mail address: [naveof@bgu.ac.il](mailto:naveof@bgu.ac.il) (O. Nave).

**Nomenclature**

$A$	pre-exponential rate factor ( $\text{s}^{-1}$ )
$B$	universal gas constant ( $\text{J k mol}^{-1} \text{K}^{-1}$ )
$C$	molar concentration ( $\text{k mol m}^{-3}$ )
$c$	specific heat capacity ( $\text{J kg}^{-1} \text{K}^{-1}$ )
$E$	activation energy ( $\text{J k mol}^{-1}$ )
$L$	liquid evaporation energy (i.e., latent heat of evaporation, enthalpy of evaporation) ( $\text{J kg}^{-1}$ )
$n$	number of droplets per unit volume ( $\text{m}^{-3}$ )
$P(\cdot)$	probability density function (PDF), also defined as $P_R$ , and for dimensionless form as $\tilde{P}_r$
$Q$	combustion energy ( $\text{J kg}^{-1}$ )
$q$	heat flux ( $\text{W m}^{-2}$ )
$R$	radius of droplet (m)
$r$	dimensionless radius
$T$	temperature (K)
$t$	time (s)
$t_{\text{react}}$	characteristic reaction time (s) defined in Eq. (3.14)

*Greek symbols:*

$\alpha$	dimensionless volumetric phase content
$\beta$	dimensionless reduced initial temperature (with respect to the so-called activation temperature $E/B$ )
$\gamma$	dimensionless parameter that represents the reciprocal of the final dimensionless adiabatic temperature of the thermally insulated system after the explosion has been completed
$\epsilon_i$	$i = 1, \dots, 3$ dimensionless parameters defined in Eq. (3.14)
$\eta$	dimensionless fuel concentration
$\theta$	dimensionless temperature
$\lambda$	thermal conductivity ( $\text{W m}^{-1} \text{K}^{-1}$ )
$\mu$	molar mass ( $\text{kg kmol}^{-1}$ )
$\rho$	density ( $\text{kg m}^{-3}$ )
$\sigma$	Stefan–Boltzmann constant ( $\text{W m}^{-2} \text{K}^{-4}$ )
$\tau$	dimensionless time
$\psi$	represents the internal characteristics of the fuel (the ratio of the specific combustion energy and the latent heat of evaporation) defined in Eq. (3.14) and for diesel fuel $\psi \gg 1$
$\Upsilon$	dimensionless parameters defined in Eq. (3.14)

*Subscripts:*

$c$	convection
$d$	liquid fuel droplets
$f$	combustible gas component of the mixture
$g$	gas mixture
$l$	liquid phase
$p$	under constant pressure
$r$	radiation
$s$	saturation
$0$	initial state

to that of the previous stage of the deformation. Finally, when  $p = 1$  the model of equations takes the original form of the equations and the final stage of deformation gives the desired solution. In 1992, Liao [5] proposed a generalization of the HPM called homotopy analysis method (HAM). The advantage of the HAM as compared to other perturbation methods is that the HAM is independent on of a small/large physical parameter (no artificial parameter is require). This method provides a simple way to control and hence to ensure the convergence of approximation series. For this purpose, one should choose a proper value of the so called convergence-control parameter. In addition, the HAM provides a great freedom to choose base function that span the solution (linear or nonlinear problem) [7]. Another advantage of the HAM is that one can construct a continuous mapping of an initial guess approximation to the exact solution of the given problem through an auxiliary linear operator. We refer the reader to [8] for more details.

In this paper we introduce a new method based on the HAM [9,10] and MIM to investigate the problem of thermal explosion in two-phases polydisperse combustible mixture of gas with fuel droplets. We present two algorithms that describe 1: the HAM technique and, 2: the combination of HAM and MIM which is our new method for  $\gamma \neq 0$ ,  $\gamma^2 = 0$  and both

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