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One-dimensional analysis for magneto-thermo-mechanical response in a functionally graded annular variable-thickness rotating disk

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ABSTRACT

In this paper, the magneto-thermo-mechanical response of a functionally graded magneto-elastic material (FGMM) annular variable-thickness rotating disk is investigated. The material properties namely material stiffness, heat conduction coefficient, thermal expansion coefficient, mass density and magnetic permeability are assumed to vary continuously along the radial direction according to a power law. The thickness profile of the disk placed in a uniform magnetic field and subjected to the thermal load is assumed to be hyperbolic in nature. The effects of the magnetic field, grading index and geometric nonlinearity on the mechanical and thermal stresses of the disk are investigated. For a specific value of the grading index the maximum radial stress due to magneto-mechanical load in a mounted FGMM disk with hyperbolic convergent profile is found away from the center. This result is different from other thickness profile disks where the radial stresses are always at the center. It is observed that unlike radial stress in a mounted FGM disk subjected to mechanical load only where it is always tensile, the radial stress due to magneto-thermal load in a mounted FGMM disk can be both tensile and compressive type. It is seen that a decrease in the value of grading index invokes shifting of the location of the maximum temperature in FGMM disk with hyperbolic convergent profile towards the outer surface of the disk.

1. Introduction

Smart or intelligent materials such as the piezoelectric and piezomagnetic materials have been extensively investigated recently due to their ability of converting energy from one form to another [1]. These materials are upgraded materials which are designed using the concept and properties of functionally graded materials (FGMs). These materials generally called functionally graded magneto-elastic materials (FGMMs) are so designed that their properties may continuously vary under the influence of magnetic field, thermal load and mechanical load [2–4]. Increased interest in magneto-thermo-elasticity in recent years can be attributed to many interesting studies on coupled magneto-thermo-mechanical behaviors of smart

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structures (see [5] and the references therein).These materials mainly constructed to operate in high temperature environments find their applications in aerospace, chemical laboratories, turbine rotors, magnetic storage elements, magnetic structural elements [6,7]. In modern technology, these materials are used in the electronic instrumentation, sensing and actuating devices [8].

FGMM plates have been used in many practical applications, such as magnetostrictive materials, optics, magnetoelastic sensors, development of highly sensitive superconducting magnetometer, supersonic airplanes, rockets and missiles etc. [9,10]. Pan and Han [11] presented an exact solution of FGMM plate under simply supported conditions. They employed the propagator matrix method for multilayered plate structure. Yun et al. [12] used series solution method and analyzed the bending of FGMM circular plates by applying layer wise model while the transverse loads were expanded in Fourier–Bessel series. There exist some other studies which concern dynamic response. Yu and Wu [13] studied wave propagation in FGMM curved plate by using Legendre orthogonal polynomials and series expansion approach and showed the effects of the piezoelectricity and piezomagnetism. Dai et al. [14] examined the dynamic characteristics of a simply supported FGMM plate by deriving 3D coupled equations based on the Hamilton principle. Chih-Ping and Yi-Chu [15] developed a modified Pagano method for the 3D dynamic responses of simply supported multilayered FGMM plates with three different lateral surface conditions. They studied the effect of the power law exponent on the natural frequencies of the FGMM plates.

Dai and Fu [16] analyzed the FGMM hollow structures subjected to mechanical load assuming related material properties represented by power law of the structure's wall thickness. Wang and Dong [17] used a theoretical method based on finite Hankel integral transforms for analyzing magneto-thermo-elastic response in FGMM cylinder. They also studied perturbation of the magnetic field vector in a conducting non-homogeneous thermo-elastic cylinder subjected to thermal shock. Paul and Narasimhan [18] studied the problem of axisymmetric axial stress wave generation in a FGMM cylindrical bar in the presence of an applied magnetic field assuming the surface of the cylinder free from mechanical load and thermal radiation. Lee [19] analyzed the FGMM circular shells subjected to magnetic and temperature fields by using Laplace transform and finite difference methods.

Thermo-mechanical stresses in circular structures such as rotating disks have been studied in [20–22]. Both types of disks i.e. disks with uniform thickness [23] and disks with variable-thickness [24–27] have been considered in the literature. It has been reported (see [28–31]) that stresses in rotating annular or solid disks with variable thickness are much lower than those with uniform thickness at the same angular velocity. Thermo-elastic analysis of FG rotating disks has been considered in [32–37].

The present study focuses on the magneto-thermo-elastic analysis of an FG disk with variable thickness under plane stress condition, symmetry with respect to the axis and the mid-plane and under thermal loading due to heat source. To the best of the authors' knowledge, no such study has been reported in the literature till date and this provides motivation for the work of this paper. More specifically, the present paper aims to study the effect of some basic factors such as magnetic field, property gradation and the geometry of the disk on stress and displacement fields in the hollow disk under free–free and fixed-free boundary conditions. The exact solutions for the non-dimensional temperature distribution and the displacement field would also be given.

2. Gradation relation

The material properties and the thick profile of the disk considered in this paper are of the same forms as have been considered by the authors in their earlier studies [38,39]. The generic forms for material properties P and thickness-profile h have the following representations:

$$P(r) = P_0\left(\frac{r}{r_0}\right)^{\varsigma}, \tag{1a}$$

$$h(r) = h_o\left(\frac{r}{r_o}\right)^{-m}. \tag{1b}$$

In (1a) and (1b), various entities have the following definitions:

r: The radius of the disk.
r_o: The outer radius of the disk.
$P(r)$: The material property of the disk at radial position r.
P_o: The material property of the disk at the outer radius.
ς: Power law index of the material dependent on the material and geometric properties of the disk but independent of r.
$h(r)$: The thickness of the disk profile at radial position r.
h_o: The thickness of the profile at the outer surface.
m: The geometric parameter that can be positive or negative representing the order of the hyperbola.

In this paper, Poisson's ratio v will be assumed to be constant and other properties of the material such as thermal conductivity k, the elastic modulus E, the density ρ, the magnetic permeability μ and the coefficient of the thermal expansion α will be taken following representation of (1a) as