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Enhanced interactive satisficing method via alternative tolerance for fuzzy goal programming with progressive preference

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This paper proposes an enhanced interactive satisficing method via alternative tolerance for fuzzy goal programming with progressive preference. The alternative tolerances of the fuzzy objectives with three types of fuzzy relations are used to model progressive preference of decision maker. In order to improve the dissatisficing objectives, the relaxed satisficing objectives are sacrificed by modifying their tolerant limits. By means of attainable reference point, the auxiliary programming is designed to generate the tolerances of the dissatisficing objectives for ensuring feasibility. Correspondingly, the membership functions are updated or the objective constraints are added. The Max–Min goal programming model (or the revised one) and the test model of the M-Pareto optimality are solved lexicographically. By our method, the dissatisficing objectives are improved iteratively till the preferred result is acquired. Illustrative examples show its power.

1. Introduction

Recently, multi-objective optimization (MOO) has become more and more important [1–3]. However, multiple objectives are conflicting, noncommensurable and imprecise, which usually results in the ultimate goal of the MOO problem becoming to seek a most preferred solution rather than the optimal one. As a powerful tool for MOO, goal programming (GP), initially introduced by Charnes and Cooper [4], is generally used in the real world decision making problems [5]. By GP, several objectives are considered simultaneously in finding a set of acceptable solutions through minimizing the deviations from the expected values. However, determining precisely the goal value of each objective is difficult for decision maker (DM). Moreover, possibly only partial information can be obtained since much of decision making takes place in an environment where the objectives, constraints or parameters are not known precisely [6–10]. In the early works, although research on randomness is devoted to uncertainty of MOO [11], there is a qualitatively different type of imprecision which cannot be equated with probability. Then, it may result in fuzzy understanding to the optimization problem for DM. Consequently, Bellman and Zadeh [12] introduce the fuzzy set theory into the traditional decision making problem. Thereafter, Narasimhan [13] initially formulates fuzzy goal programming (FGP) for specifying imprecise aspiration levels of the goals in a fuzzy environment. It attracts many researches [14–17].

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In MOO, preference refers to the opinion of DM concerning points in criterion space. According to the way the DM articulates or incorporates, the preferences can be categorized into prior, posterior, and progressive articulation. For FGP, preference is very important, and the final decision is dependent on it. Generally, prior articulation of preference includes importance and priority in FGP. Tiwari et al. [18] use the weighted or hierarchical additive model to handle the importance and priority preference among the objectives. Chen and Tsai [19] model the importance difference and priority levels by means of the desirable satisficing degree comparisons and the satisficing degree comparisons. Hu et al. [20,21] propose the varying-domain method and enhanced additive priority optimization model to solve MOO with preemptive priorities respectively. Additionally, Li and Hu [22,23] present the satisficing optimization approaches by holding the more important objective achieving the higher desirable satisficing degree for the vague importance requirement.

In most situations, however, little a priori knowledge and experience about preference are known about a decision making problem in advance [24,25]. This type of preference is called progressive preference. It requires DM continually to provide input during the running of an algorithm. Since the utility function may be not available, interactive methods become interesting and desirable. During the interactive process, DM attends to the algorithm and is supported to investigate what is achievable and what should be done in order to arrive at the most preferred solution. Generally, the interactive optimization techniques are categorized into two classes: an interactive optimization method [26] and an interactive satisficing method [27]. For FGP with progressive preference, there are a few approaches [28–30]. They require DM give the reference membership degrees at each step. However, these methods only acquire the local solutions since strong assumptions must be ensured or repeated comparisons by interpolation and extrapolation are required. Although the structure of the membership function has been used by Werners [31,32] to model the progressive preference, possibly there is no feasible solution for the arbitrary regulation of the tolerances. Thus, Hu et al. [33,25] have adopted the principle of the step method (STEM) to handle fuzzy goals with inequality fuzzy relations. In FGP, the equality fuzzy relation is also important. It is difficult to deal with this relation when progressive preference is required. Therefore, three types of fuzzy relations are considered simultaneously in this paper. Based on the approaches of Hu et al. [33,25], the enhanced interactive satisficing optimization method for solving FGP with progressive preference is proposed. During solving, some satisficing objectives are relaxed iteratively to improve the dissatisficing objectives till all the objectives are satisficing to DM. The progressive preference is realized by means of alternative tolerances of the objectives, which is gradually elicited by interaction with DM but not given beforehand. In order to guarantee its feasibility with respect to the alternative tolerances, the attainable reference point method [27] is referred to in this paper, and the auxiliary programming is introduced to determine the new tolerances of the three types of fuzzy relations. If the new reference values are not preferred to the aspiration levels, they are regarded as the tolerant limits such that the membership functions are reconstructed. Otherwise, the objective constraints are added as a special case. With the new membership functions and objective constraints, the Max–Min GP or the revised GP is formulated, and the test model for ensuring the M-Pareto optimality is also proposed. They are solved lexicographically. In this paper, the tolerances are changed during iterations even if the satisficing degrees of the newly constructed membership functions are low. By the proposed method, the preferred result can be acquired.

The rest of this paper is organized as follows. In Section 2, the formulation and the corresponding definitions of FGP are given. Section 3 describes the enhanced interactive satisficing optimization method via alternative tolerance. The algorithm step is presented in Section 4. Section 5 analyzes the convergence of the proposed method. The effectiveness of our approach is demonstrated by the numerical examples and practical application example in Section 6. Section 7 draws the conclusions.

2. Fuzzy goal programming

In general GP, DM can give the precise target values of the objectives from which unwanted deviations are minimized. In a fuzzy environment, however, it is difficult for DM to present the accurate goals. DM can only gives the vague target values with different fuzzy relations where all objectives are required to approach as nearly as possible even to be superior to the aspiration levels. Then, the FGP problem is written as follows [9,12,34]:

$$\begin{aligned} &\textit{Find}: && \boldsymbol{x} \\ &\textit{Such that} && f_i(\boldsymbol{x}) \rightarrow f_i^*,\ i=1,\ldots,k \\ & && \boldsymbol{x} \in G = \{\boldsymbol{x} | g_j(\boldsymbol{x}) \leqslant 0,\ j=1,\ldots,m\}, \end{aligned} \tag{1}$$

where $f_i(\boldsymbol{x})$, $(i=1,\ldots,k)$ are the objectives; and G represents system constraints. f_i^* is the aspiration level associated with the objective function $f_i(\boldsymbol{x})$. '$\rightarrow$' denotes the fuzziness of f_i^*, and it involves three types of fuzzy relations, i.e. '$\tilde{\leqslant}$' (fuzzy minimization), '$\tilde{\geqslant}$' (fuzzy maximization) and '$\tilde{=}$' (fuzzy equality). Correspondingly, the sets including the different objectives with different fuzzy relations are defined using the following symbols:

S: the set consisting of all of the objectives, written as the set of index, i.e. $\{1,\ldots,k\}$.
$S^{\leqslant}$: the set consisting of the objectives with fuzzy relation '$\tilde{\leqslant}$', where $S^{\leqslant} \subseteq S$.
$S^{\geqslant}$: the set consisting of the objectives with fuzzy relation '$\tilde{\geqslant}$', where $S^{\geqslant} \subseteq S$.
$S^{=}$: the set consisting of the objectives with fuzzy relation '$\tilde{=}$', where $S^{=} \subseteq S$.