



On front solutions of the saturation equation of two-phase flow in porous media



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ABSTRACT

We investigate the existence of “front” solutions of the saturation equation of two-phase flow in porous media. By front solution we mean a monotonic solution connecting two different saturations. The Brooks–Corey and the van Genuchten models are used to describe the relative-permeability – and capillary pressure–saturation relationships. We show that two classes of front solutions exist: self-similar front solutions and travelling-wave front solutions. Self-similar front solutions exist only for horizontal displacements of fluids (without gravity). However, travelling-wave front solutions exist for both horizontal and vertical (including gravity) displacements. The stability of front solutions is confirmed numerically.

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1. Introduction

The two-phase flow process in porous media occurs in many environments. Typical examples of such process in porous media include water–oil and oil–gas displacements in natural petroleum reservoirs [1, 2], brine and CO₂ movement in geologic formations such as petroleum reservoirs and deep saline aquifers [3], water and air flow in natural surface/subsurface unsaturated soils [4, 5], and water and air or oxygen displacement in the underground during aerated subsurface drip irrigation or oxidation [6, 7]. In the theory of two-phase flow in porous media the system of equations which describes the displacement of a fluid by another fluid can be reduced to a nonlinear advection–diffusion equation which can be written as a conservation law of the quantity of one fluid contained in a given soil volume. The resulting equation is called the saturation equation. The advective term is due to injection and/or gravitation while the diffusive term is due to capillarity (the force due to the difference on pressures of the two fluids). Note that the “diffusion-like” behaviour due to capillarity is different from the standard molecular diffusion in multicomponent miscible flow.

One of the characteristics of the saturation equation is the existence of semi-analytical solutions. These semi-analytical solutions are presented as self-similar solutions and travelling-wave solutions. Since it seems impossible to list here all of the publications on this subject, we will refer the reader to the fundamental book of [8] for further details.

In this paper, we are interested in self-similar and travelling-wave solutions of the saturation equation which are front solutions. By front solution we mean a wave connecting two regions where the saturations (fluid contents) are different. From mathematical point of view, a front solution is a monotonic function $\Psi(\xi)$ in the $\xi - \Psi$ plane (where ξ and Ψ represent distance and saturation, respectively) that is defined for all ξ , and such that Ψ connects two saturations Ψ_1 at $-\infty$

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(i.e. $\lim_{\xi \rightarrow -\infty} \Psi(\xi) = \Psi_l$) and Ψ_r at ∞ (i.e. $\lim_{\xi \rightarrow \infty} \Psi(\xi) = \Psi_r$) and stabilizes at $\pm\infty$ (i.e. $\lim_{\xi \rightarrow \pm\infty} \Psi'(\xi) = 0$). Such solutions represent the asymptotic of a wide class of other more general solutions that correspond to various initial conditions.

The well-known Brooks–Corey and van Genuchten models which describe relative-permeability and capillary pressure curves are used in this paper. We found that when self-similar solutions exist (i.e. the case of horizontal displacements: advection and diffusion without gravitation), front solutions exist for any couple of boundary saturation values independently from the model. When travelling-wave solutions of the first type exist (i.e. the case of vertical displacements: advection and diffusion with gravitation), front solutions exist for both Brooks–Corey and van Genuchten models. However, such front solutions does not exist for any couple of boundary saturation values. The shape of the advective flux function plays an essential role on the determination of the admissible boundary saturations. When capillarity is the sole mechanism of transport (i.e. diffusion only without advection), travelling-wave front solutions of the second type exist for some alternative models (i.e. a combination of both Brooks–Corey and van Genuchten models). A numerical study shows that travelling-wave front solutions are stable for the case of vertical displacements whereas self-similar fronts are stable for the case of horizontal displacements.

2. Mathematical model

The partial differential equations which describe the flow of the two incompressible and immiscible fluids in the porous medium are based on the conservation of mass of the two phases

$$\phi \frac{\partial S_\alpha}{\partial t} + \frac{\partial q_\alpha}{\partial x} = 0 \quad (1)$$

and the modified Darcy's laws

$$q_\alpha = -\frac{kk_{r\alpha}(S_\alpha)}{\mu_\alpha} \left(\frac{\partial p_\alpha}{\partial x} + \rho_\alpha g \right), \quad (2)$$

where $\alpha = n$ for the non-wetting phase and $\alpha = w$ for the wetting phase. In these equations x is distance and t is time, S_α is the saturation of phase α (the fractional volume of the pore space filled with phase α), q_α , p_α , $k_{r\alpha}$, ρ_α and μ_α are the flow rate, the pressure, the relative-permeability, the density and the viscosity of phase α , respectively. The parameters ϕ and k represent the porosity and permeability of the porous medium and g is the acceleration due to gravity.

The two-phase saturations are constrained by

$$S_n + S_w = 1. \quad (3)$$

Combining the two-mass conservation Eq. (1) and using Eq. (3) yields the expression of the total flux

$$q_n + q_w = q(t). \quad (4)$$

The difference between the non-wetting and wetting phase pressures defines the capillary pressure

$$p_c = p_n - p_w. \quad (5)$$

Capillary pressure function in porous media is a function of saturation [9]. Eq. (3) allows to write the relative-permeability and the capillary pressure functions as functions of one of the two saturations. In this paper we shall write these relations as functions of the non-wetting phase saturation, $k_{r\alpha} = k_{r\alpha}(S_n)$ and $p_c = p_c(S_n)$.

Using Eqs. (2), (4) and (1) for $\alpha = n$ we obtain after some mathematical manipulations the saturation equation (see for instance [10])

$$\phi \frac{\partial S}{\partial t} + \frac{\partial}{\partial x} \left[F(S) \left(\frac{\mu_w q}{kk_{rw}(S)} + (\rho_w - \rho_n)g - \frac{\partial p_c(S)}{\partial x} \right) \right] = 0, \quad (6)$$

where $F(S) = \frac{k}{\mu_w} \frac{k_{rn}(S)k_{rw}(S)}{k_{rn}(S)+Mk_{rw}(S)}$ and $M = \frac{\mu_n}{\mu_w}$ is the viscosity ratio. In Eq. (6) we omitted index “n” and now $S \equiv S_n$ is the non-wetting phase saturation. The saturation Eq. (6) is a nonlinear advection–diffusion transport equation. The advective flux of the non-wetting phase which is due to injection and gravity effects is defined by

$$A(S) = F(S) \left(\frac{\mu_w q}{kk_{rw}(S)} + (\rho_w - \rho_n)g \right). \quad (7)$$

The diffusive flux is due to capillarity effects which generate the following diffusivity of the non-wetting phase

$$D(S) = F(S) \frac{dp_c(S)}{dS}. \quad (8)$$

Substituting (7) and (8) into (6) we obtain the saturation equation of the non-wetting phase written in a simple form as follows

$$\phi \frac{\partial S}{\partial t} + \frac{\partial}{\partial x} \left(A(S) - D(S) \frac{\partial S}{\partial x} \right) = 0. \quad (9)$$

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