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Minimum quench energies of uncooled low temperature superconductors with temperature dependent thermophysical parameters

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An analytical method for calculation of minimum quench energies (MQEs) of uncooled composite low temperature superconductors is presented. The method takes into account transient heat transfer in the conductor as well as temperature dependent ohmic heat generation and temperature dependent thermophysical properties of the conductor. MQE of the conductor is calculated based on the analysis of evolution of peak temperature in the normal zone. The method is validated by comparison of the obtained results with the experimental data as well as with analytical and numerical results taken from literature.

1. Introduction

A composite superconductor carrying a transport current can experience energy disturbances of various mechanisms. These disturbances can result in the appearance of normal zones that, depending on the disturbance energy, either disappear or grow infinitely. The minimum energy of the heat disturbance required to destroy the superconductivity, in the limiting case of δ-like perturbations, is called the minimum quench energy (MQE) or the critical energy. MQE is one of the most important characteristics of the stability of a superconducting composite. Its value depends on such factors as: the transport current, external magnetic field, thermal and electric properties of the composite, its cross-sectional geometry, stabilizer to superconductor ratio, cooling conditions.

The value of the MQE for a specific conductor and operating conditions can be obtained either experimentally or calculated by numerical simulations at different levels of approximation and complexity [1–12]. The stability experiments and the numerical simulations most often involve a time-consuming trial-and-error procedure. A trial disturbance energy is selected and the resulting transient is monitored until the quench or recovery can be observed. Such a procedure is repeated for various values of the disturbance energy until the just quenching and the just not quenching case are identified and the value of MQE is assessed with the required accuracy.

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Nomenclature

A	total area of cross-section of conductor, m^2
B	external magnetic field, T
C	heat capacity of composite superconductor, J/(m^3 K)
c_p	specific heat of composite superconductor, J/(kg K)
d	density, kg/m^3
E	energy of heat disturbance, J
E_c	minimum quench energy, (critical energy, MQE), J
f	volume fraction of normal metal
$G(T)$	ohmic heat generation per unit lateral surface area of conductor, W/m^2
G_i	heat dissipation by disturbance per unit lateral surface per unit time, W/m^2
$G_{\max}$	maximum value of ohmic heat generation, i.e. ohmic heat generation with whole current in stabilizer, W/m^2
i	reduced current defined as $i = I/I_c$
I	transport current, A
I_c	critical current at $T = T_0$, A
j_c	critical current density, defined as $j_c = I_c/A$, A/m^2
k	thermal conductivity of conductor, W/(m K)
$2l$	length of conductor subjected to heat disturbance, m
L	Laplace transform
P	perimeter of conductor, m
QE	quench energy, J
RRR	residual resistivity ratio
t	time, s
t_i	duration of heat disturbance, s
T	temperature, K
T_c	critical temperature at $I = 0$, K
T_{c1}	critical temperature at given current, i.e. temperature where current sharing starts, K
T_0	initial temperature of conductor, K
T^*	variable defined by Eq. (6), K^2
x	co-ordinate along conductor, m
X	dimensionless co-ordinate along conductor

Greek letters

α	dimensionless ohmic heat generation with whole current in stabilizer
ε	dimensionless energy of heat disturbance
ε_c	dimensionless MQE
φ	dimensionless parameter defined as $\varphi = \alpha/(1 + \Theta^*_{c1})$
Λ	Lorentz number, V^2/K^2
Θ^*	dimensionless temperature defined by Eq. (8)
$\Theta^*_1(0,\tau)$	dimensionless maximum temperature in normal zone
Θ^*_{c1}	dimensionless critical temperature at given current
ρ	electrical resistivity of conductor, Ω m
τ	dimensionless time
τ_i	dimensionless duration of heat disturbance
ψ	dimensionless capacity of the heat source resulting from local heat disturbance

Subscripts

Cu	copper
NbTi, n	NbTi in normal state
NbTi, s	NbTi in superconducting state

Analytical methods are useful tools to assess approximate values of MQE prior performing costly experiments or complex numerical modeling. The results obtained with simplified analytical models provide a reference point for testing and validation of new developed numerical codes. Fast analytical methods can be particularly useful at the early phase of the design of a superconducting device. At this stage several strands and conductor design concepts are usually considered and the comparative parametric analysis is performed in which relatively wide ranges of parameters characterizing a conductor are scanned. Such initial analysis is aimed at selecting the most promising candidates for further more detailed experimental