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Unrelated parallel machines scheduling with deteriorating jobs and resource dependent processing times

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ABSTRACT

We consider unrelated parallel machines scheduling problems involving resource dependent (controllable) processing times and deteriorating jobs simultaneously, i.e., the actual processing time of a job is a function of its starting time and its resource allocation. Two generally resource consumption functions, the linear and convex resource, were investigated. The objective is to find the optimal sequence of jobs and the optimal resource allocation separately. This paper focus on the objectives of minimizing a cost function containing makespan, total completion time, total absolute differences in completion times and total resource cost, and a cost function containing makespan, total waiting time, total absolute differences in waiting times and total resource cost. If the number of unrelated parallel machines is a given constant, we show that the problems remain polynomially solvable under the proposed model.

1. Introduction

There are various situations in which the job processing times may be increase due to delays or waiting of jobs, such kinds of tasks are called deteriorating jobs. Job deterioration (time-dependent scheduling) appears, for instance, in the steel production where the temperature of an ingot drops below a certain level while waiting to enter a rolling machine, which requires reheating of the ingot before rolling. Similar situations will also occur in maintenance planning and scheduling, national defense or cleaning assignments, where any delay in processing a job is penalized by incurring additional time for accomplishing the job. An extensive survey of different scheduling theory and models involving deteriorating jobs (time-dependent scheduling) can be found in Gawiejnowicz [1]. Huang and Wang [2] considered scheduling problems with deteriorating jobs, i.e., the model $p_j = a_j + bt$, where p_j, a_j and t, respectively, represent the actual processing time, normal (basic) processing time and the starting time of job J_j on a single machine, and b represents the common deterioration rate. For single machine, parallel identical machines and unrelated machines, they proved that the total absolute differences in completion (waiting) times minimization can be solved in polynomial time (i.e., $1|p_j = a_j + bt|TADC(TADW)$, $Pm|p_j = a_j + bt|TADC(TADW)$, $Rm|p_j = a_j + bt|TADC(TADW)$, where C_j (W_j) is the completion (waiting) time of job J_j, $TADC = \sum_{i=1}^{n}\sum_{j=i}^{n}|C_i - C_j|$ and $TADW = \sum_{i=1}^{n}\sum_{j=i}^{n}|W_i - W_j|$). Jafari and Moslehi [3] considered the same model with Huang and Wang [2], for the number of tardy jobs minimization problem $(1|p_j = a_j + bt|\sum_{j=1}^{n}U_j)$, they proposed a branch-and-bound

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algorithm. Wang and Wang [4] considered a three-machine permutation flow shop problem with deteriorating jobs, in which the actual processing time of job J_j on machine M_i is $p_{ij} = a_{ij} + bt$, where a_{ij} is a normal (basic) processing time of job J_j on machine M_i. For the makespan minimization scheduling problem (i.e., $F3|prmu,\ p_{ij} = a_{ij} + bt,\ b \geqslant 0|C_{\max}$), they proposed two heuristic algorithms and a branch-and-bound algorithm.

On the other hand, scheduling problems with controllable processing times (resource-dependent processing times) in which the actual job processing time is assumed to be a function of the amount of resource allocated have been extensively studied. More recent surveys of different scheduling models and problems involving controllable processing times (resource-dependent processing times) were given by Shabtay and Steiner [5], and Edis et al. [6]. Shabtay et al. [7] considered a single machine group scheduling problem with a family due date assignment method. They proved that a cost function consisting of earliness, tardiness and due date assignment penalties problem can be solved in polynomial time if all jobs belonging to group technology assumption. They also extended the results to the case of controllable processing times for both linear and convex resource consumption functions (i.e., the models $p_j = a_j - \theta_j u_j$ (where $\theta_j \geqslant 0$ and u_j is the amount of a non-renewable resource allocated to job J_j) and $p_j = \left(\frac{a_j}{u_j}\right)^k$ ($k > 0$ is a constant, and $u_j > 0$ is the amount of a non-renewable resource allocated to job J_j)). Xu et al. [8] considered a single machine total tardiness minimization scheduling problem with a convex controllable processing times, i.e., the model $p_j = \left(\frac{a_j}{u_j}\right)^k$. They proved that the problem $1|p_j = \left(\frac{a_j}{u_j}\right)^k, \sum_{j=1}^n T_j \leqslant T, d_j = d|\sum_{j=1}^n u_j$ can be solved in $O(n^2)$ time, where d_j, C_j and $T_j = \max\{C_j - d_j\}$ is the due date, the completion time and the tardiness of job J_j, respectively. Wang and Wang [9] considered the problem $1|p_j = \left(\frac{a_j}{u_j}\right)^k,\ \sum_{j=1}^n w_j C_j \leqslant F|\sum_{j=1}^n u_j$, where w_j is the weight of the job J_j and F is the total weighted flow time of a given permutation. In order to solve this problem, they proposed a heuristic algorithm and a branch-and-bound algorithm.

It is natural to study scheduling problems combining deteriorating jobs and controllable processing times (resource allocation). However, to the best of our knowledge, there exist only a few papers dealing with the resource allocation and deteriorating jobs simultaneously. Wei et al. [10] discussed single-machine scheduling problems considering linear resource dependent processing times and deteriorating jobs concurrently, i.e., $p_j = a_j + bt - \theta_j u_j$, where a_j, b, t, θ_j and u_j all have the same meaning as before. They proved that the problems $1|p_j = a_j + bt - \theta_j u_j|\delta_1 C_{\max} + \delta_2 TC + \delta_3 TADC + \delta_4 \sum_{j=1}^n G_j u_j$ and $1|p_j = a_j + bt - \theta_j u_j|\delta_1 C_{\max} + \delta_2 TW + \delta_3 TADW + \delta_4 \sum_{j=1}^n G_j u_j$ can be solved in polynomial time, where $TC = \sum_{j=1}^n C_j,\ TW = \sum_{j=1}^n W_j, G_j$ denote the per time unit cost associated with the resource allocation of job J_j and $\delta_1 \geqslant 0, \delta_2 \geqslant 0, \delta_3 \geqslant 0$ and $\delta_4 \geqslant 0$ denote the weights. Wang and Wang [11] discussed single-machine scheduling problems considering convex resource dependent processing times and deteriorating jobs concurrently, i.e., $p_j = \left(\frac{a_j}{u_j}\right)^k + bt, u_j > 0$. They proved that the problems $1|p_j = \left(\frac{a_j}{u_j}\right)^k + bt|\delta_1 C_{\max} + \delta_2 TC + \delta_3 TADC + \delta_4 \sum_{j=1}^n G_j u_j$ and $1|p_j = \left(\frac{a_j}{u_j}\right)^k + bt|\delta_1 C_{\max} + \delta_2 TW + \delta_3 TADW + \delta_4 \sum_{j=1}^n G_j u_j$ can be solved in polynomial time. Wang and Wang [12] considered the problems $1|p_j = a_j + bt - \theta_j u_j|\sum(\alpha E_j + \beta T_j + \gamma d_j + G_j u_j)$ and $1|p_j = \left(\frac{a_j}{u_j}\right)^k + bt|\sum(\alpha E_j + \beta T_j + \gamma d_j + G_j u_j)$. For three popular due date assignment methods, they proposed a polynomial time algorithm, respectively. The phenomena resource allocation and time-dependent processing times (deteriorating jobs) occurring simultaneously can be found in steel production, i.e., in the process of preheating ingots by gas to prepare them for hot rolling on the blooming mill. Before the ingots can be hot rolled, they have to achieve the required temperature. However, the preheating time of the ingots depends on their starting temperature. The preheating time can be shortened by the increase of the gas flow intensity, i.e., the more gas is consumed, the shorter lasts the preheating process. Thus, the ingot preheating time depends on the starting moment of the preheating process and the amount of gas consumed during it (Wei et al. [10], and Bachman and Janiak [13]). However, in the realistic production settings, there is rarely a single-machine environment, i.e., parallel machine scheduling problems are more practical in industrial production environments, such as electronic industry, mechanical industry and so on. Therefore, in this paper, we consider unrelated parallel machines scheduling with linear (convex) resource dependent processing times and deteriorating jobs at the same time. The rest of this paper is organized as follows. Notations and assumptions are given in Section 2. In Sections 3 and 4, we show that the problems can be solved in polynomial time for linear and convex resource dependent processing times, respectively. Section 5 are devoted to examples. In Section 6, conclusions are presented.

2. Problem formulation

Given n independent and non-preemptive jobs $J = \{J_1, J_2, \ldots, J_n\}$ and m unrelated parallel machines $M = \{M_1, M_2, \ldots, M_m\}$ (each job could be processed by a free machine) with associated the actual processing time p_{ij} ($i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n$). All the jobs are available for processing at time zero. In this research, we consider the following models:

A time and linear resource dependent processing times model (Wei et al. [10]):

$$p_{ij} = a_{ij} + bt - \theta_{ij} u_{ij}, \tag{1}$$