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Review of wavelet methods for the solution of reaction–diffusion problems in science and engineering

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ABSTRACT

Wavelet method is a recently developed tool in applied mathematics. Investigation of various wavelet methods, for its capability of analyzing various dynamic phenomena through waves gained more and more attention in engineering research. Starting from 'offering good solution to differential equations' to capturing the nonlinearity in the data distribution, wavelets are used as appropriate tools at various places to provide good mathematical model for scientific phenomena, which are usually modeled through linear or nonlinear differential equations. Review shows that the wavelet method is efficient and powerful in solving wide class of linear and nonlinear reaction–diffusion equations. This review intends to provide the great utility of wavelets to science and engineering problems which owes its origin to 1919. Also, future scope and directions involved in developing wavelet algorithm for solving reaction–diffusion equations are addressed.

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1. Introduction

Nonlinear partial differential equations (PDEs) of reaction–diffusion type arise in many applications which include physical sciences, developmental biology, ecology, physiology, finance, to name a few. Reaction–diffusion systems are usually coupled systems (multiple numbers) of parabolic partial differential equations. In applications to population biology, the reaction term models growth, and the diffusion term accounts for migration. Some of them are models for pattern formation in morphogenesis, for predator–prey and other ecological systems, for conduction in nerves, for epidemics, for carbon monoxide poisoning, and for oscillating chemical reactions.

Reaction–diffusion systems are usually coupled systems of parabolic partial differential equations which include pattern formation in morphogenesis, for predator–prey and other ecological systems, for conduction in nerves, for epidemics, for carbon monoxide poisoning, oscillating chemical reactions, pulse splitting and shedding, reactions and competitions in excitable systems and stability issues. Reaction–diffusion equations (RDEs) in their simplest form are written as

$$U_t = \frac{\partial U}{\partial t} = D\frac{\partial^2 U}{\partial x^2} + f(U), \tag{1}$$

where $U = U(x, t)$ is the vector of dependent variables, $f(U)$ is a non-linear vector-valued function of u (the reaction term), and D is the diffusion coefficient. The reaction term arise from any interaction between the components of u. The parameter u may be a vector of predator–prey interactions, competition or symbiosis. The diffusion terms may represent molecular diffusion or some "random" movement of individuals in a population [1].

The reaction–diffusion system may be extended to reaction–diffusion–convection type given by

$$\frac{\partial U}{\partial t} = U_t = f(U) + D\frac{\partial^2 U}{\partial x^2} + C\frac{\partial U}{\partial x}, \tag{2}$$

where C is the convection coefficient.

It is known that for reaction–diffusion systems [involving reaction terms $\frac{\partial^2 U}{\partial x^2}$], the numerical treatment of the reaction terms is influential on the numerical results.

2. Derivation of reaction–diffusion equations (RDEs)

The mechanism of diffusion models is the movement of many individuals in an environment or media. The particles reside in a region, which we call Ω is open set of R^n (the $n-$ th dimensional space with Cartesian coordinate system) with $n \geqslant 1$. The derivation of reaction–diffusion equation is well documented in the link http://www.resnet.wm.edu/~jxshix/math490/lecture-chap1.pdf

The diffusion coefficient $D(x)$ is not a constant in general since the environment is usually heterogeneous. But when the region is approximately homogeneous, we can assume that $D(x) = D$, the above equation can be simplified to

$$\frac{\partial P}{\partial t} = D\Delta P + f(t, x, P), \tag{3}$$

where $\Delta P = div(\nabla P) = \sum_{i=1}^{n} \frac{\partial^2 P}{\partial x_i^2}$ is the Laplacian operator.

3. Importance of reaction–diffusion problems in engineering

3.1. Civil engineering

In recent years, the study of the aggregate alkali reaction in fluid leaching processes is of special interest in analysis of concrete dams in civil engineering. The numerical findings carried out here are directed towards a better understanding of the model. It is well documented in Ref. [2].

3.2. Chemical engineering

The solution of reaction–diffusion systems, however, is a very challenging task because the equations are time-dependent, often highly nonlinear, and coupled. Efficient solution techniques which can characterize many parameter combinations are therefore important. Furthermore, in many applications – such as chemical engineering – understanding,