Applied Mathematical Modelling 38 (2014) 828–838

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Interval weight generation approaches for reciprocal relations

Meimei Xia [a,*], Zeshui Xu [b]

[a] School of Economics and Management, Tsinghua University, Beijing 100084, China
[b] Institute of Sciences, PLA University of Science and Technology, Nanjing 210007, China

ARTICLE INFO

Article history:
Received 3 February 2012
Received in revised form 10 June 2013
Accepted 9 July 2013
Available online 20 July 2013

Keywords:
Reciprocal relation
Incomplete reciprocal relation
Interval reciprocal relation
Multiplicative consistency
Additive consistency
Goal programming

ABSTRACT

In this paper, we propose methods to derive interval weight vectors from reciprocal relations for reflecting the inconsistency when decision makers provide preferences over alternatives (or criteria). Several goal programming models are established to minimize the inconsistency based on multiplicative and additive consistency, respectively. Especially, if we obtain a crisp weight vector from a reciprocal relation, then it is consistent. Then, we extend the proposed methods to incomplete reciprocal relations and interval reciprocal relations and develop the corresponding models to derive interval weight vectors. Several examples are also given to compare the developed methods with the existing ones.

1. Introduction

In multi-criteria decision making problems, decision makers usually use preference relations (e.g. reciprocal relations or multiplicative preference relations) to express their preferences over each pair of alternatives (or criteria). To get the priority vector of preference relations, a lot of weight-determining methods have been developed in the last decades. Wang et al. [1] proposed a chi-square method (CSM) for obtaining priority vectors from multiplicative and reciprocal relations. Fan et al. [2] proposed a goal programming approach to solving group decision making problems where the preference information on alternatives provided by the decision makers is represented in two different formats, i.e. multiplicative preference relations and reciprocal relations. Xu [3] defined the concepts of incomplete reciprocal relation, additive consistent incomplete reciprocal relation and multiplicative consistent incomplete reciprocal relation, and then proposed two goal programming models, based on additive consistent incomplete reciprocal relation and multiplicative consistent incomplete reciprocal relation, respectively, for obtaining the priority vector of incomplete reciprocal relation. Wang and Fan [4] applied the logarithmic and geometric least squares methods (LLSM and GLSM) to deal with group decision analysis problems with reciprocal relations, where multiplicative preference relations, if any, are transformed into reciprocal relations through proper transformation technique. Xu (2005) proposed a least deviation method (LDM) to obtain a priority vector of a reciprocal relation using the transformation relation between multiplicative preference relation and reciprocal relation.

Because of the uncertainty of real problems and intuitiveness of human judgments, it often happens that the given comparisons are inconsistent each other and some of them are missing. The priority vector obtained by the above methods are always crisp values, however, the provided preference relations are always inconsistent due to the decision makers' intuition, and interval evaluations are more suitable for representing uncertain information. Entani and Tanaka [5] and Sugihara

* Corresponding author. Tel.: +86 18810491647.
E-mail addresses: meimxia@163.com (M. Xia), xuzeshui@263.net (Z. Xu).

0307-904X/$ - see front matter © 2013 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.apm.2013.07.018

et al. [6] proposed methods to obtain interval weights from multiplicative preference relations. They denoted that even if preference values with respect are given as crisp values, the priority weights should be estimated as intervals because of a decision maker's uncertainty of judgments. Guo and Tanaka [7] proposed linear programming and quadratic programming problems to minimize the imprecision of judgment to estimate interval probabilities of multiplicative preference relations. Entani and Sugihara [8] proposed models to obtain interval priority of the multiplicative preference relations from the viewpoints of entropy in probability, sum or maximum of widths, or ignorance. The interval priority helps a decision maker to recognize his/her uncertainty on the problem as well as the relative importance of the attributes. However these methods are not suitable to deal with reciprocal relations. Motivated by this idea, we give two methods to determine the interval priority vectors from reciprocal relations based on multiplicative and additive consistency, and then extend them to incomplete and interval situations.

To do this, the remainder of this paper is organized as follows: in Section 2, we first give some basic concepts. In Section 3, some models are established to derive interval weights from reciprocal relations based on multiplicative and additive consistency, respectively, and then are extended to deal with the incomplete reciprocal relations. Section 4 constructs several models to determine interval weights for interval reciprocal relations and incomplete interval reciprocal relations. Section 5 gives some concluding remarks.

2. Some basic concepts

Definition 1 [9]. Let $X = \{x_1, x_2, \ldots, x_n\}$ be a set of alternatives, then $A = (a_{ij})_{n\times n}$ is called a reciprocal relation on $X \times X$ with the condition that

$$a_{ij} \geqslant 0,\ a_{ij} + a_{ji} = 1, \quad i,j = 1,2,\ldots,n, \tag{1}$$

where a_{ij} denotes the degree that the alternative x_i is prior to the alternative x_j.

If some elements in A cannot be given by the decision maker, then A can be called an incomplete reciprocal relation [3], in which we denote the unknown values by "–", and the others provided by the decision maker satisfy the condition (1).

Definition 2 [10]. Let $A = (a_{ij})_{n\times n}$ be a reciprocal relation, then A is called a multiplicative consistent reciprocal relation if it satisfies the following property:

$$a_{ij} > 0,\ a_{ij}a_{jk}a_{ki} = a_{ji}a_{kj}a_{ik}, \quad i,j,k = 1,2,\ldots,n \tag{2}$$

and such a reciprocal relation can also be given by

$$a_{ij} = \frac{w_i}{w_i + w_j}, \quad i,j,k = 1,2,\ldots,n, \tag{3}$$

where $w = (w_1, w_2, \ldots, w_n)^{\mathrm{T}}$ is the priority vector of A and

$$\sum_{i=1}^{n} w_i = 1,\ w_i > 0, \quad i = 1,2,\ldots,n. \tag{4}$$

Definition 3 [10]. Let $A = (a_{ij})_{n\times n}$ be a reciprocal relation, then A is called an additive consistent reciprocal relation, if the following is satisfied:

$$a_{ij} = a_{ik} + a_{kj} - 0.5, \quad i,j,k = 1,2,\ldots,n, \tag{5}$$

which can also be given as [11]:

$$a_{ij} = 0.5(w_i - w_j + 0.5), \quad i,j,k = 1,2,\ldots,n, \tag{6}$$

where $w = (w_1, w_2, \ldots, w_n)^{\mathrm{T}}$ is the priority vector of A and satisfy the condition (4).

For an incomplete reciprocal relation, if its known elements satisfy the condition (3), then it is considered as a multiplicative consistent incomplete reciprocal relation; if its known elements satisfy the condition (6), then it is considered as an additive consistent incomplete reciprocal relation.

Definition 4 [12]. Let $\widetilde{a} = [a^-, a^+]$ and $b = [b^-, b^+]$ be two any interval values, where $0 \leqslant a^- \leqslant a^+ \leqslant 1$ and $0 \leqslant b^- \leqslant b^+ \leqslant 1$, then the degree of possibility of $\widetilde{a} \geqslant \widetilde{b}$ is defined as:

$$d\left(\widetilde{a} \geqslant \widetilde{b}\right) = \frac{\max\{0, a^+ - b^-\} - \max\{0, a^- - b^+\}}{a^+ - a^- + b^+ - b^-}. \tag{7}$$