Applied Mathematical Modelling 38 (2014) 864–877

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Application of a heterogenous multiscale method to multi-batch driven pipeline

Sašo Blažič [a,*], Gerhard Geiger [b], Drago Matko [a]

[a] University of Ljubljana, Faculty of Electrical Engineering, Tržaška 25, SI-1000 Ljubljana, Slovenia
[b] Westfälische Hochschule Gelsenkirchen, Fachbereich Elektrotechnik und Angewandte Naturwissenschaften, Neidenburger Str. 43, 45897 Gelsenkirchen, Germany

ARTICLE INFO

Article history:
Received 10 January 2011
Received in revised form 2 June 2013
Accepted 2 July 2013
Available online 22 July 2013

Keywords:
Pipeline
Multiscale simulation
Lumped parameter model
Method of characteristics

ABSTRACT

The problem of simulating pipelines that are used for transporting different fluids is addressed in the paper. The model of the multi-batch pipeline is obtained by extending the classical "water hammer equations" (dealing with pressure and velocity of the medium) with fluid density. In such way a system of nonlinear partial differential equations is derived and solved by the method of characteristics. However, the ordinary differential equations resulting from the method of characteristics are defined on domains with very different slopes in the (x, t) space. A heterogenous multiscale method using two grids is capable of coping with associated numerical problems as shown by comparison of simulated and measured data on a real pipeline.

1. Introduction

Real time transient model (RTTM) based leak monitoring systems [1–3] require a sophisticated mathematical model of the flow in pipelines. The so called "water hammer equations" are relatively simple mathematical models assuming isentropic flow; they are obtained using the principles of mass and momentum conservation [4,5]. These models are based on a one-dimensional approach for one-phase liquid flow. Up to now, there is no analytical solution for this non-linear, partial differential equation system available. However, numerical techniques have traditionally been exploited to solve such problems. Several different approaches exist in the literature for tackling the aforementioned problem. Very often the method of characteristics [6] has been used, other approaches include finite volume [7], finite difference (explicit [8] or implicit [9,10]), finite element methods [11], polynomial differential quadrature [12], and transfer function modelling [13]. All these methods represent a part of the field of Computational Fluid Dynamics (CFD). The increased computational power of modern digital computer has steadily increased the range of possibilities in that field.

Not only RTTM based leak monitoring systems have been widely studied in the recent years, but also a handful of commercial products for RTTM based leak monitoring systems is offered. An excellent literature review of RTTM based leak detection methods can be found in [14]. It has been shown [15] that water-hammer wave attenuation, shape and timing parameters may be significantly affected by violating the idealised conditions of the water-hammer equations. Some approaches also dealt with turbulent flows [16,17] where vaned pipe bends are analysed in [17]. Yet, very little has been published on the simulation of multi-batch driven pipelines, where different fluids are transported through the same pipeline. A simple model of such pipelines was presented in [18].

* Corresponding author. Address: Tržaška 25, 1000 Ljubljana, Slovenia. Tel.: +386 1 4768763; fax: +386 1 4264631.
E-mail addresses: saso.blazic@fe.uni-lj.si (S. Blažič), gerhard.geiger@fh-gelsenkirchen.de (G. Geiger), drago.matko@fe.uni-lj.si (D. Matko).

0307-904X/$ - see front matter
http://dx.doi.org/10.1016/j.apm.2013.07.001

The purpose of the present paper is to propose an alternative solution for simulation of multi-batch driven pipelines. This is achieved by extending the method of characteristic to multi-batch driven pipelines and by applying the heterogenous multi-scale method for the simulation of equations which are not defined on the same domain.

In Section 2 the simple water hammer equation model will be extended to the case when different fluids are transported through the same pipeline. This enables the description of multi-product flows with multiple products or batches being transported at the same time in one pipeline. The same model was used in [18], where the problem was linearised and solved analytically. In this paper, however, the method of characteristics [6] is applied to the extended model as shown in Section 3. The ordinary differential equations resulting from the method of characteristics are defined on domains with very different slopes in the (x, t) space. In order to circumvent this problem, a heterogenous multiscale method using two grids is proposed in Section 4. This method is capable of coping with associated numerical problems as shown in Section 5 presenting comparison of simulated and measured data on a real pipeline.

2. Mathematical model of the pipeline

The classical solution for unsteady flow problems is obtained by using the equations for continuity, momentum and energy. These equations correspond to the physical principles of mass, momentum and energy conservation. By applying these equations, a coupled non-linear set of partial differential equations is obtained that is very difficult to solve analytically. To date, there is no general closed-form solution. Further problems arise in the case of turbulent flow, which introduces stochastic flow behavior. Therefore, the mathematical derivation for the flow through a pipeline is a mixture of both theoretical and empirical approaches.

The following assumptions for the derivation of a mathematical model of the flow through pipelines are made:

1. *Fluid is compressible.* Compressibility of fluid results in an unsteady flow.
2. *Flow is viscous.* Viscosity causes shear stresses in a moving fluid.
3. *Flow is adiabatic.* No transfer of energy between fluid and pipeline will be considered.
4. *Flow is isothermal.* Temperature changes due to pressure changes can be neglected for liquids. Under these circumstances, temperature changes could only be result of friction effects, but these effects will also be neglected. Therefore, the temperature along the pipeline is supposed to be constant.
5. *Flow is one-dimensional.* All characteristics of the pipeline such as velocity v and pressure p depend only on the x-axis laid along the pipeline.

Consider now a pipeline of length L_p and with constant diameter

$$D = D(x) = 2R = const. \tag{1}$$

The continuity equation in conservative form for the one-dimensional case yields [11]

$$\frac{d\rho}{dt} + \rho\frac{\partial v}{\partial x} = 0, \tag{2}$$

with density $\rho(x,t)$, velocity $v(x,t)$, and with the substantial or total derivative

$$\frac{d\rho}{dt} \equiv \frac{\partial\rho}{\partial t} + v\frac{\partial\rho}{\partial x}. \tag{3}$$

The Momentum Equation in conservative form for the one-dimensional case yields [11]

$$\rho\frac{dv}{dt} = -\rho g\sin\alpha - \frac{\partial p}{\partial x} + \frac{\partial p_L}{\partial x}, \tag{4}$$

with pressure $p(x,t)$. The quantity $g\sin\alpha$ is the x-component of the standard gravity vector $\mathbf{g}$. The pressure loss p_L rely on the shear stress τ_R. The formula from Darcy and Weisbach [19] states that

$$\frac{\partial p_L}{\partial x} = -\rho\frac{\lambda v \mid v \mid}{2D}, \tag{5}$$

with the dimensionless friction coefficient $\lambda(v)$. This equation holds for the laminar flow as well as for the turbulent flow. Laminar flow is described by [19]

$$\lambda = \lambda(v) = \frac{64}{Re}, \tag{6}$$

if the dimensionless Reynolds number

$$Re = \frac{D}{\nu}\cdot v \tag{7}$$