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An interface crack moving between magnetoelectroelastic and functionally graded elastic layers

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ABSTRACT

A constant crack moving along the interface of magnetoelectroelastic and functionally graded elastic layers under anti-plane shear and in-plane electric and magnetic loading is investigated by the integral transform method. Fourier transforms are applied to reduce the mixed boundary value problem of the crack to dual integral equations, which are expressed in terms of Fredholm integral equations of the second kind. The singular stress, electric displacement and magnetic induction near the crack tip are obtained asymptotically and the corresponding field intensity factors are defined. Numerical results show that the stress intensity factors are influenced by the crack moving velocity, the material properties, the functionally graded parameter and the geometric size ratios. The propagation of the moving crack may bring about crack kinking, depending on the crack moving velocity and the material properties across the interface.

1. Introduction

Composites made of piezoelectric/piezomagnetic materials exhibit magnetoelectric effect that is not present in single-phase piezoelectric or piezomagnetic materials, and such magnetoelectric materials have potential applications as transducers, acoustic/ultrasonic devices, and magnetic and/or electric sensors. Studies on the properties of piezoelectric/piezomagnetic composites have been carried out by numerous investigators [1,2]. In particular, there is a growing interest among researchers in solving fracture mechanics problems in media possessing coupled piezoelectric, piezomagnetic and magnetoelectric effects, that is, magnetoelectroelastic effects. The crack initiation behavior in magnetoelectroelastic composite under in-plane deformation was investigated by Song and Sih [3]. Gao et al. [4,5] presented exact treatments on the crack problems in magnetoelectroelastic solids. Wang and Mai [6] considered the mode III crack problems in an infinite piezoelectromagnetic medium using complex variable technique, and the same authors gave an exact analysis for mode III cracks between two dissimilar magnetoelectroelastic layers [7]. Hu and Li [8] studied the crack in a magnetoelectroelastic strip under longitudinal shear. Dynamic response of cracked magnetoelectroelastic medium has been investigated by Li [9] and Feng et al. [10]. Rokne et al. [11] investigated the problem of a moving anti-plane shear crack in a piezoelectric layer bonded to dissimilar elastic infinite spaces. Kwon and Lee [12] studied the moving interfacial crack between piezoelectric ceramic and elastic layers. Moving crack problems in magnetoelectroelastic materials have been solved by Hu and Li [13], Hu et al. [14] and Tapholme [15], respectively. Most recently, the pre-kinking of a moving crack in a magnetoelectroelastic material has been analyzed by Hu and Chen [16].

In recent years, functionally graded materials (FGMs) have been widely applied in extreme loading environment, and the fracture mechanics of FGMs have attracted much attention [17,18]. The problems of moving crack in FGMs have been

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investigated by Wang and Meguid [19], Jiang and Wang [20], Singh et al. [21], Cheng and Zhong [22], Bagheri and Ayatollahi [23] and Yan and Jiang [24]. The idea of FGM has been extended to piezoelectric and magnetoelectroelastic materials due to the multi-fields coupling effects and potential applications in manufacturing smart devices, and the fracture mechanics of functionally graded magnetoelectroelastic materials (FGMMs) have become an area of increasing interest. Hu et al. [25] considered an anti-plane shear crack in a functionally gradient piezoelectric interlayer between dissimilar piezoelectric half planes. Ueda [26] studied the dynamic response of a central crack in a functionally graded piezoelectric strip under electromechanical impact. Feng et al. [27] considered the problem of multiple cracks on the interface between a piezoelectric layer and an orthotropic substrate. Matbuly [28] investigated the multiple crack propagation along the interface of a non-homogeneous composite subjected to anti-plane shear loading.

Mode III crack problems in FGMMs have been investigated by Feng and Su [29], Ma et al. [30], Li and Lee [31] and Chen and Chue [32]. Zhou and Chen [33] obtained the basic solution of a mode-I limited-permeable crack in functionally graded piezoelectric/piezomagnetic materials. Ma and Lee [34] gave a theoretical analysis of in-plane problem in functionally graded nonhomogeneous magnetoelectroelastic bimaterials. Under the assumption of the in-plane magneto-electro-mechanical loadings, Zhong and Lee [35] studied the dielectric crack problem for a magnetoelectroelastic strip with functionally graded properties.

Although crack problems in magnetoelectroelastic material or functionally graded materials have been investigated by many researchers, the moving crack along the interface between magnetoelectroelastic and functionally graded elastic layers has not been investigated. The objective of this paper is to study the moving interface crack between magnetoelectroelastic and functionally graded elastic layers under anti-plane mechanical loading and in-plane electric and magnetic loading. Fourier transforms are applied to reduce the mixed-boundary-value problem to dual integral equations, which can be converted into Fredholm integral equations. The field intensity factors are defined and the crack kinking phenomena are observed; the effect of material properties, crack moving velocity, geometric size, and functionally graded parameter are analyzed.

2. Basic equations

Consider a linear magnetoelectroelastic material which is assumed to be transversely isotropic and denote the Cartesian coordinates of a point by $X_j (j = 1, 2, 3)$, i.e., $X_1 = X,\ X_2 = Y,\ X_3 = Z$.

The dynamic equilibrium equations are given as

$$\sigma_{ij,i} + f_j = \rho \frac{\partial^2 u_j}{\partial t^2}, \quad D_{i,i} - f_e = 0, \quad B_{i,i} = 0 \tag{1}$$

where σ_{ij}, u_j, D_i and B_i are the components of stress, displacement, electrical displacement and magnetic induction, respectively; f_j and f_e are the body force and electric charge density, respectively; ρ is the mass density of the magnetoelctroelastic material; a comma followed by i $(i = 1, 2, 3)$ denotes partial differentiation with respect to the coordinate X_i, and the summation convention over repeated indices is applied. The constitutive equations can be written as

$$\begin{aligned} \sigma_{ij} &= c_{ijks}\varepsilon_{ks} - e_{sij}E_s - h_{sij}H_s \\ D_i &= e_{iks}\varepsilon_{ks} + \lambda_{is}E_s + \beta_{is}H_s \quad , \\ B_i &= h_{iks}\varepsilon_{ks} + \beta_{is}E_s + \gamma_{is}H_s \end{aligned} \tag{2}$$

where ε_{ks}, E_s and H_s are the components of strain, electric field and magnetic field, respectively; c_{ijks}, e_{iks}, h_{iks} and β_{is} are the elastic, piezoelectric, piezomagnetic and electromagnetic constants, respectively; λ_{is} and γ_{is} are dielectric permitivities and magnetic permeabilities, respectively. The following reciprocal symmetries hold:

$$\begin{aligned} &c_{ijks} = c_{jiks} = c_{ijsk} = c_{ksij}, \quad e_{sij} = e_{sji}, \\ &h_{sij} = h_{sji}, \quad \beta_{ij} = \beta_{ji}, \quad \lambda_{ij} = \lambda_{ji}, \quad \gamma_{ij} = \gamma_{ji}. \end{aligned} \tag{3}$$

The gradient equations are

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad E_i = -\phi_{,i}, \quad H_i = -\varphi_{,i}. \tag{4}$$

where ϕ and φ are the electric and magnetic potential, respectively.

The governing equations simplify considerably if we consider only the out-of-plane displacement, and the in-plane electric and magnetic fields, i.e.,

$$u_X = u_Y = 0, \quad u_Z = u_Z(X, Y, t), \tag{5}$$

$$E_X = E_X(X, Y, t), \quad E_Y = E_Y(X, Y, t), \quad E_Z = 0, \tag{6}$$

$$H_X = H_x(X, Y, t), \quad H_Y = H_Y(X, Y, t), \quad H_z = 0, \tag{7}$$