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Accurate vibration analysis of skew plates by the new version of the differential quadrature method

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ABSTRACT

An accurate free vibration analysis of skew plates is presented by using the new version of the differential quadrature method (DQM). Eight combinations of simply supported (S), clamped (C) and free (F) boundary conditions are considered. Detailed solution procedures are given and key points for success by using the DQM are emphasized. A way to simplifying the programming in using the DQM is proposed. Convergence study is made for the simply supported skew plate with a large skew angle. Good convergence of frequencies is observed. The DQ results agree very well with the existing first known accurate upper bound solutions, obtained by using Ritz method taking into considerations of the bending stress singularities occurred at corners having obtuse angles. Since slight discrepancy between the DQ data and the known accurate solutions is observed for plates with large skew angles, the DQ results are also compared with data obtained by using finite element method with very fine meshes to verify their accuracy.

1. Introduction

Skew plate is one of the common structural elements in many kinds of high-performance surface and air vehicle. For example, they are used in the construction of wings, tails, and fins of swept-wing aircrafts, missiles. Skew plates are also used as panels in skew bridges. Therefore their structural behavior is important to structural engineers. A vast body of literature [1–11] exists on the free vibration of thin and thick skew plates by using various methods. Research work on this topic has been well documented by Zhou and Zheng [8].

Due to the complicated mathematical structure of the differential equations, it is not an easy task to obtain closed form solutions for skew plates. It is even not an easy task to obtain accurate fundamental frequency when the skew angle is large, since strong stress singularity at the obtuse plate corners exists [1,2,11]. Leissa [3] emphasized the importance to consider the existing of the singularity since it will strongly affect the frequencies. Without doing so, poor convergence of frequencies may be obtained when only polynomial displacement trial functions are used in the Ritz method, but the upper bound convergence of the solution improves considerably when the hybrid trial sets of polynomials and corner functions are simultaneously utilized [1]. Therefore, difficulty would be encountered in obtaining accurate frequency by using various continuum-base and numerical methods if the singularity is not considered. This might be one of the main reasons why only a few results are reported in the open literature for plates with large skew angles although extensive studies have been made on the free vibration of skew plates.

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The first known accurate frequencies are given by Leissa and his co-workers [1,2,11] for thin skew plates and by Liew et al. [4] for thick skew plates. However, the accuracy of the frequencies has not been well addressed in the open literature [8]. It is noted that obtaining the accurate frequencies is not an easy task, since matrix ill-conditioning may be encountered if more corner functions are used [1,2]. Therefore, other reliable and accurate methods are also desirable.

Recently, accurate results are obtained by using the moving least square Ritz (MLS-Ritz) method [8]. Different number of MLS-Ritz points is used and most results are smaller but very close to the accurate upper bound solutions reported in [2]. It is noted that, however, for the CSCS (C-clamped and S-simply supported) and CSSS plate with skew angle of 75°, the third to fifth mode frequencies are quite different from the data listed in [2]. Besides, the third MLS-Ritz mode frequency of CSCS skew plate is even larger than the upper bound solution. It is mentioned that the convergence of the MLS-Ritz is not monotonic with the increase of the number of points. Thus the question arises which frequency is more accurate since discrepancy exists.

The differential quadrature method (DQM) is a relatively new technique [12,13] to solve the static and free vibration problems of anisotropic rectangular plate. The method can yield very accurate results with relatively much less computational effort [14] and is a very promising approach for the vibration analysis of plates [15]. Therefore, DQM is to be used to obtain accurate frequencies of skew plate, since the problem shows mathematically analogy to the free vibration of anisotropic rectangular plates. The key to success in using the DQM is the accurate way to apply the multiple boundary conditions [16] and the usage the non-uniform grid spacing [17]. Thus, the new version of the differential quadrature method (DQM) proposed by the authors [18,19] is to be used for obtaining accurate frequencies of skew plates. Seven combinations of simply supported (S) and clamped (C) boundary conditions and one with free (F) boundary conditions are considered for skew angles varying from 15° to 75°. To show the solution accuracy, comparisons between the proposed DQ method and conventional ones are made via numerical results. Two conventional DQ methods, one with modified δ-method to applying boundary conditions (DQM-δ) and the other which directly couples the discretized boundary conditions with the discretized governing equations [20] (DQM-CBCGE) are involved for comparisons. Detailed formulations and solution procedures are given. To overcome the difficulty in programming, a simple way is proposed. Convergence study is made for simply supported rhombic plate with skew angle of 60°. To investigate the solution accuracy, the DQ results are compared with the upper bound solutions [1,2,11], MLS-Ritz data [8] or results obtained by NASTRAN with very fine meshes.

2. Theory and method of analysis

Basic equations for the free vibration analysis of isotropic thin skew plates are presented in this section. The solution procedures by using the new version of DQM are also given. The difference of the new version of the DQM from the conventional DQM and key points for success in obtaining the accurate results are emphasized.

2.1. Free vibration of the thin isotropic skew plate

Consider the free vibration of a thin isotropic skew plate with side length a and b, and thickness t, as shown in Fig. 1. The material parameters, E, ν, ρ, are the elasticity modulus, Poisson's ratio, and the mass density, respectively. The relations between the skew coordinates ξ, η and the Cartesian coordinates x, y are

$$x = \xi + \eta \sin\theta; \quad y = \eta / \cos\theta, \tag{1}$$

where θ is the skew angle shown in Fig. 1.

Then the governing differential equation for free vibration of the skew plate can be written in the skew coordinate system as [18]

$$D\frac{\partial^4 w}{\partial \xi^4} + 4D(-\sin\theta)\frac{\partial^4 w}{\partial \xi^3 \partial \eta} + 2D(1 + 2\sin^2\theta)\frac{\partial^4 w}{\partial \xi^2 \partial \eta^2} + 4D(-\sin\theta)\frac{\partial^4 w}{\partial \xi \partial \eta^3} + D\frac{\partial^4 w}{\partial \eta^4} = \cos^4\theta \rho t \omega^2 w, \tag{2}$$

where $D = Et^3/12(1 - \nu^2)$ is the bending rigidity of the plate, w is the deflection, and ω is the circular frequency.

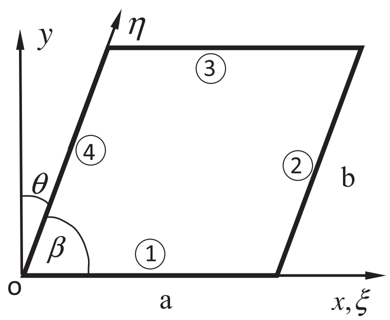

Fig. 1. Sketch of a skew plate.