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An analytical dynamic model for single-cracked beams including bending, axial stiffness, rotational inertia, shear deformation and coupling effects

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ABSTRACT

This paper develops an analytical dynamic model for cracked beams including bending, axial stiffness, rotational inertia, shear deformation and the coupling of the last two effects. The damage is modelled using a rotational spring that simulates the crack based on fracture mechanics theory. The developed model is used to predict variations on natural frequencies for several crack sites and damage magnitude along the beam. The importance of this work lies in the development of an analytical model that has no approximation due to discretization of the displacement field. This initial theoretical approach describes the expected behaviour for changes in the natural frequencies for simply-supported and clamped-free beams with the precision that only analytical methods allow. The results provide a useful benchmark to compare with approximate numerical methods that can be used to model and analyse the problem. The model showed similar results for long span beams, but the inclusion of rotational inertia and shear deformation effects rendered improvements in the dynamic behaviour mainly in the case of slender and short span beams when compared with the simplified Euler–Bernoulli model.

1. Introduction

Experimental research on the effects of crack and damage on the integrity of structures has been performed by several authors in the last decades. Liebowitz and Claus [1], one of the pioneers in the area, have studied the load capacity of columns with notches and suggested a failure criterion based on stress concentration factor to explain differences in load capacity from non-notched columns. Chondros [2,3] and Christides and Barr [4] should be mentioned as authors that extended the views of the previous work from the load capacity behaviour to a dynamic point of view, including in their work comparisons with experimental data. This was perceived later as a way to detect damage using inverse analysis. Shen and Pierre [5] developed a bi-dimensional finite element approach using a Galerkin approach and very good agreement with experimental data up to the third natural frequency and mode shape was obtained. Narkis [6] presented a framework to the crack identification based on the frequency response function of cracked beams. The developed equations to predict the natural frequencies changes due to crack were based on bending effects in a simply supported linear beam, and they were used to solve the inverse problem in order to find the crack position. A state of the art review presented by Dimaragonas [7] gathered hundreds of works that address the problem of vibration on cracked structures. The reviewed papers were then classified according to the way the problem is addressed. Since then, another vast quantity of papers has been published in the referred subject.

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The development of an analytical model for dynamic modelling of beams has a great importance since no approximation due to discretization is included in the formulation. This leads to unbiased values that can be used as benchmark to compare new numerical models.

Chondros et al. [8,9] presented a continuous cracked bar vibration theory for rods with cracks and compared the obtained first natural frequency with experimentally measured ones in cracked rods. A good correlation with the experimental results was obtained. Problems related to the extent of the crack undermined some of the experimental results for crack depth to height ratios greater than 0.5. Fernandez-Saez and Rubio [10] suggested a closed form solution for the problem of obtaining the first natural frequency of a cracked simply supported beam using the Rayleigh method. It was compared the analytical results with those obtained with a finite element model, resulting in excellent correlation. Unfortunately, no experimental data was used to validate and the proposed formulation is only useful for the first natural frequency.

Narkis [6] works, Owolabi and Swamidas [11] proposed the detection of cracks in beams using FRF in order to determine their extents and locations. The damage detection schemes used in their study depends on the measured changes in the first three natural frequencies and the corresponding amplitudes of the measured acceleration FRF. Chondros [12], using the fracture mechanics theory, developed a model based on the local flexibility at the cracked section. They developed a continuous flexibility function to represent the crack based on the displacement fields in the vicinity of the crack and then fitted the obtained results (just for the first natural frequency) with numerical and experimental ones.

General methods were proposed by [13–15] to obtain the eigenfrequencies and mode shapes of beams containing multiple cracks and subjected to axial forces. Cracks were assumed to introduce changes in the local flexibility and they were modelled as rotational springs. The method used a set of end conditions as initial parameters to determine the mode shape functions. The proposed method could efficiently be used to detect crack locations and extent in numerically generated beam-columns examples based just on frequency changes. It is stated by [14] that the method could be used to predict the critical load of damaged structures based on eigenfrequency measurements.

Naniwadekar et al. [16] suggested a technique that uses experimentally measured natural frequency changes of horizontal steel hollow pipes to predict crack location, depth and orientation. They modelled the crack by a rotational spring with a straight front in different orientations in a section of rod specimen. In their paper the rotational spring stiffness for a crack size and orientation was obtained experimentally by the deflection and the vibration methods. They report that the proposed method is very robust, since the obtained maximum variation in damage location was 2.68%, which was much less than the induced change in stiffness.

Most of the papers that are found in the literature deals with the crack detection problem and frequency estimation assuming just the bending behaviour of the cracked beams [4,8–10,13,11], Aydin [15], and Naniwadekar et al. [16] and some few include important effects like axial force effects [14]. The presented paper extends these analyses considering some other important effects in the beam dynamic behaviour. For those interested in experimental data related to this theme, it is suggested the following papers: Christides and Barr [4], Shen and Pierre [5,17,18], Rizos and Aspragathos [19], Ruotolo and Surace [20], Chondros et al. [8,9,12], Chondros [12], Saavedra and Cuitiño [21], Owolabi and Swamidas [11], Nahvi and Jabbari [22], Chen et al. [23], Naniwadekar et al. [16].

This paper presents a general framework to analyse the changes in natural frequencies in simply supported and clamped-free beams including some important behaviours that should be considered when dealing of beams of diverse dimensions. It is proposed to compare two models for the dynamic vibration equation: (a) one model including bending and axial stiffness effects (Simplified Model) similar to that presented by [14] and (b) a model including bending, buckling, rotational inertia, shear deformation and couplings effects (Complete Model). These models are compared using simple beam examples for long and short spans including one cracked site in order to verify the importance of such terms in the governing equations. The idea is to analyse the models regarding their accuracy in evaluating frequency changes in these situations.

2. Equations for cracked beams including bending, buckling, shear deformation, rotational inertia and coupling effects

The general equation for beams including bending, shear deformation and rotational inertia can be improved, based on the equation proposed by [24] including the axial stiffness effect and coupling of rotational inertia and shear deformation. This formulation is written as indicated by Eq. (1).

$$\underbrace{EI\frac{\partial^4 v}{\partial x^4}+\bar{m}\frac{\partial^2 v}{\partial t^2}}_{bending}+\underbrace{N\frac{\partial^2 v}{\partial t^2}}_{buckling}-\underbrace{\bar{m}r^2\frac{\partial^4 v}{\partial x^2\partial t^2}}_{rotational\ inertia}+\underbrace{\frac{EI}{k'AG}\frac{\partial^2}{\partial x^2}\left(-\bar{m}\frac{\partial^2 v}{\partial t^2}\right)}_{shear\ deformation\ \ effect}+\underbrace{\frac{\bar{m}r^2}{k'AG}\frac{\partial^2}{\partial t^2}\left(\bar{m}\frac{\partial^2 v}{\partial t^2}\right)}_{coupling\ of\ \ rotational\ inertia\ and\ shear\ deformation}=0, \tag{1}$$

where E is the Young modulus, G is the Shear modulus, I is the moment of inertia, A is the sectional area, N is the axial force acting in the beam axis, $\bar{m}$ is the mass per unit length, r is the radius of gyration, $k'A$ is the effective shear area of the cross section (k' = 5/6 for rectangular sections), x is the coordinate space along beam axis v is the displacement in y direction and t is the time variable.

In order to solve Eq. (1), assuming that time and position variables can be separated, one can write the following general solution in the time and space domain:

$$v(x,t)=Y(x)e^{i\omega t}. \tag{2}$$