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A fuzzy algorithm for continuous capacitated location allocation model with risk consideration

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ABSTRACT

This paper presents a continuous capacitated location-allocation model with fixed cost as a risk management model. In the presented model, the fixed cost consists of production and installation costs. The model considers risk as percent of unsatisfied demands. The fixed cost is assigned to a zone with a predetermined radius from its center. Because of uncertain environment, demand in each zone is investigated as a fuzzy number. The model is solved by a fuzzy algorithm based on α-cut method. After solving the model based on different α-values, the zones with the largest possibilities are determined for locating new facilities and the best locations are calculated based on the obtained possibilities. Then, the model is solved based on different α-values to determine best allocation values. Also, this paper proposes a Cross Entropy (*CE*) algorithm considering multivariate normal and multinomial density functions for solving large scale instances and is compared with *GAMS*. Finally, a numerical example is expressed to illustrate the proposed model.

1. Introduction

Location-allocation (LA) problem is to locate a set of new facilities such that the transportation cost from these facilities to customers is minimized and an optimal number of facilities have to be placed in an area of interest in order to satisfy the customer demand. This problem occurs in many practical settings such as the determination and location of warehouses, distribution centers, communication centers and production facilities. Since LA problem was proposed by Cooper [1] and spread to a weighted network by Hakimi [2], network LA problem and many models were presented in [3]. For a review, see [4–7]. For more researchers investigated capacitated LA problem see [8,9] where the capacities of facilities are limited. Most of work is done for the deterministic case and many models and algorithms were presented in [10–12]. Ref. [13] introduced a zone-dependent fixed cost uncapacitated location-allocation model within the framework of minisum location of facilities in the continuous space. They put forward efficient algorithm for determining the optimal solution for the single facility location problem. Solution methods of the continuous location-allocation problem were discussed in [14]. Heuristic methods including variable neighborhood search, tabu search and genetic algorithms, have proven to be the best way forward to tackle large problem instances. For a recent discussion of the computational aspects of this problem, refer to [15].

Assuming, there are m customers (demand points) indexed by i, n facilities indexed by j, x_j is coordinates of the facility j, w_{ij} is quantity supplied to the customer i by the facility j. $d(x_j,a_i)$ is distance between the customer i and the facility j, w_i is demand of the customer i, and s_j is capacity of the facility j. Then the model P_1 is as follows:

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$$P_1 : \min \sum_{j=1}^{n} \sum_{i=1}^{m} w_{ij} \cdot d(x_j, a_i), \tag{1}$$

S.t.

$$\sum_{j=1}^{n} w_{ij} = w_i,\ i = 1, 2, \ldots, m, \tag{2}$$

$$\sum_{i=1}^{m} w_{ij} \leqslant s_j,\ j = 1, 2, \ldots, n, \tag{3}$$

$$w_{ij} \geqslant 0,\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n. \tag{4}$$

Eq. (1) is the objective function consists of the transportation cost. Constraint set (2) is the demand constraint. Constraint set (3) guarantees the capacity constraint of each facility. Constraint set (4) is the standard constraint.

The above model and other location allocation problems do not consider fixed cost in continuous space, but, there are cases in real world which may be included some zones with high installation cost or forbidden zones and it needs to be considered with fixed cost. In this paper, we are interested in finding the location of n facilities in the continuous space with allocation of each facility to each customer in m demand points so that the total cost of transportation, installation, production and unsatisfied demands of customers is minimized. The fixed cost is assigned to a zone with a predetermined radius from its center. Also, for more adopting on real world, since demand of customers may not be certain, we consider the model in uncertain conditions and provide the final problem as a risk management model.

The reminder of the paper is organized as follows: in Section 2 a literature review about risk management models in the location problem is provided. In Section 3 we present the mathematical model. Solution approach proposing Cross Entropy (*CE*) algorithm and based on α-cut method is provided in Section 4. In Section 5 a numerical example is given to illustrate the usability of presented the model. Finally, Section 6 draws the conclusions with future researches.

2. Literature review

Several researches have been investigated risk in facility location problem. Ref. [16] considered the design and retrofit problem of a supply chain consisting of several production plants, warehouses and markets and the associated distribution systems. In order to take into account the effects of the uncertainty in the production scenario, a two-stage stochastic model is constructed. A stochastic version of the location model with risk pooling was proposed in [17]. The goal of the model is to find solutions that minimize the expected total cost including location, transportation, and inventory costs across all scenarios. They presented a Lagrangian-relaxation–based exact algorithm for the model. The capacitated warehouse location model with risk pooling was introduced by [18]. The model provides a logistics system in which a single plant ships one type of product to a set of retailers, each with an uncertain demand. Also, the model is solved by a Lagrangian relaxation solution algorithm. Ref. [19] developed a multi-objective stochastic programming approach for supply chain design under uncertainty. They used the goal attainment technique to obtain the Pareto-optimal solutions. A location-optimization problem in a stochastic environment with several risk factors was considered by [20]. The demand at each customer site was considered probabilistic and correlated with demands at the other customer sites. A new solution methodology was introduced to optimize the "Value-at-Risk" (VaR) measure in a location problem. They designed a branch-and-bound algorithm to solve the

Table 1
Comparison between the works.

Author(s)	Location model	Risk type	Space	Uncertainty
Guillen et al. [16]	Multi objective supply chain	Scenario based	Discrete	Stochastic
Snyder et al. [17]	Location with risk pooling	Scenario Based	Discrete	Stochastic
Ozsen et al. [18]	Warehouse location	Uncertain demand	Discrete	Stochastic
Azaron et al. [19]	Multi-objective stochastic	Scenario Based	Discrete	Stochastic
Wagner et al. [20]	Uncapacitated p-median	Value-at-Risk	Discrete	Stochastic
You et al. [21]	Multi-product supply chain	Uncertain demand	Discrete	Stochastic
Mete and Zabinsky [22]	Location with vehicle routing	Disaster	Discrete	Stochastic
Cui et al. [23]	Reliable facility location	Risk of disruption	Discrete	Stochastic
Liu et al. [25]	Two-echelon inventory/logistics	Stochastic demand	Discrete	Stochastic
Peng et al. [26]	Reliable logistics network design	Disruption	Discrete	Stochastic
Chen et al. [27]	Location selection	Disaster	Discrete	Judgmental
Hahn and Kuhn [28]	Single-stage supply chain	Scenario Based	Discrete	Stochastic
Wang and Watada [24]	Fuzzy facility location	Value-at-Risk	Discrete	Fuzzy
Nickel et al. [29]	Multi-stage supply chain	Scenario Based	Discrete	Stochastic
This research	LA with fixed cost	Uncertain demand	Continuous	Fuzzy