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# Proprietor and customer costs in the incomplete hub location-routing network topology

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ABSTRACT

The hub location problem finds the location of hubs and allocates the other nodes to them. It is widely supposed the network created with the hub nodes is complete in the extensive literature. Relaxation of this basic supposition forms the present work. The model minimizes the cost of the proprietor, including the fixed costs of hubs, hub links and spoke links. Costs of hub and spoke links are contemplated as fixed cost or maintenance cost. Moreover, the model considers routing costs of customers who want to travel from origins to destinations. In this study, we offer a model to the multiple allocations of the hub location problems, under the incomplete hub location-routing network design. This model is easily transformed to other hub location problems using one or more constraints. No network format is dictated on the hub network. We suggest a set of valid inequalities for the formulation. Some lower bounds are developed using a Lagrangian relaxation approach and the valid inequalities. Computational analyses evaluate the performances of the lower bounding implementations and valid inequalities. Furthermore, we explore the effects of several factors on the design and solution time of the problem formulation.



## 1. Introduction

Hub location problems have been raised in the past three decades in the location problems. Hub network is widely employed in the telecommunications [1], transportation [2], delivery systems [3] and postal services [4]. Hub location is one of the attractable problems that have been studied by many researchers and scientists. They have been proposed many articles in various models and methods to develop this problem in recent years [5].

The hub location typically involves locating hub and non-hub nodes and allocating non-hubs to hub nodes with spoke links. Hub nodes join and distribute flows when it is profitable. In fact, flows stream in the hub network in three phases: (I) Collecting: they move forward their origin nodes to the allocated hubs, (II) Shipping: flows transfer through the hub links if necessary, and (III) Distributing: flows go away from the hubs and reach to the destination nodes. In the hub location problems, generally, the objective is to minimize the total costs of collecting and flows distribution.

In a general view, hub location problems are four aspects. The first one is an objective function included median, covering, center which has been introduced well in [6]. Another objective is order median, which recently proposed by Puerto et al. [7]. The second aspect is the allocation strategy categorized into single and multiple which their effects are studied in [8]. The other trait is about the number of hubs. This number is computed by model or predefined by decision makers. If this number is predetermined, the model is called *p*-hub. Finally, in hub location problems, capacity of hubs is considered in several researches such as Ernst & Krishnamoorthy [9], Marin [10] and Correia et al. [11].

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In the classical hub location models, three main assumptions were often considered: (I) the hub network is a complete graph like Fig. 1(d), (II) using hub links includes a discount factor $0 \leqslant \alpha \leqslant 1$, and (III) direct connections between the non-hub nodes are forbidden. In this research, the first assumption is relaxed. This makes possible incomplete hub network on the model. The incomplete hub network can generally be divided into four categories: (I) tree shape, (II) ring network, (III) special form and (IV) general form. In the background's section, the papers which proposed an incomplete hub network are reviewed. In this paper, the hub network lies in the fourth category (i.e., General form). An example of a network designed by general form is given in the Fig. 1(c). In this figure, there are ten nodes including four hubs and six non-hubs. Moreover, there are five hub links and eight spoke links. Two non-hubs are allocated in a multiple manner, and four non-hub is assigned just to one hub.

One of the major applications of the incomplete hub location model is the transportation system, particularly public and urban transportation. A comparison between main component of public transportation network and incomplete hub location networks is explained as follows. Buses (taxis) lines and rapid lines such as subways and metro in public transportation play the role of spoke links and hub links in the incomplete hub location networks, respectively. Additionally, rapid line and bus (taxi) stations are hub nodes and spoke nodes, respectively. In fact, rapid line edges connect centres of each region to each other in the city, and the spoke links are used for internal connections in the regions. Furthermore, the most demanding regions regarding the mass of flow in a city are the locations where are worthy of linking directly to rapid lines.

The public and urban transportation applications of hub network are rarely formulated in the literature, since they need special network topology (i.e., general form) different from the usual hub-and-spoke network. In these applications, passenger would like to pass through the shortest path subject to the using network has the minimum fixed cost and transshipment cost [12]. Also, in passenger transportation, linking all hubs are extremely costly. Consequently, using the complete hub network is inefficient [13]. According to the classification of O'Kelly and Miller [12], the created network under the stated conditions belongs to the alleged Protocol F which includes hub location problems with multiple allocations of non-hub nodes to hubs, and hub-to-hub links. In this protocol, multiple hubs are allowed, non-hub node connection is not allowed and inter-hub connectivity is partially. This network topology also can be applied in some telecommunication systems and financial networks.

In this paper, median objective, multiple allocation strategy, undetermined number of hubs and uncapacitated characteristics are used. Subsequently, we call the model as Uncapacitated Multiple Allocation Hub Location over Incomplete Hub Network (UMAHLIHN). Moreover, we provide a formulation UMAHLIHN problem, which is suitable for public transportation system. Two perspectives are considered in this paper. The network proprietor who aims to use a network design with the minimum cost and the customer who is willing to pay the least money. UMAHLIHN minimizes the cost of the proprietor, containing the fixed costs of hubs, hub links and spoke links. Furthermore, the model considers routing costs of flows as the customer costs. A set of valid inequalities is suggested to tighten the formulation. Some lower bounds are also proposed based on a Lagrangian relaxation approach and the valid inequalities.

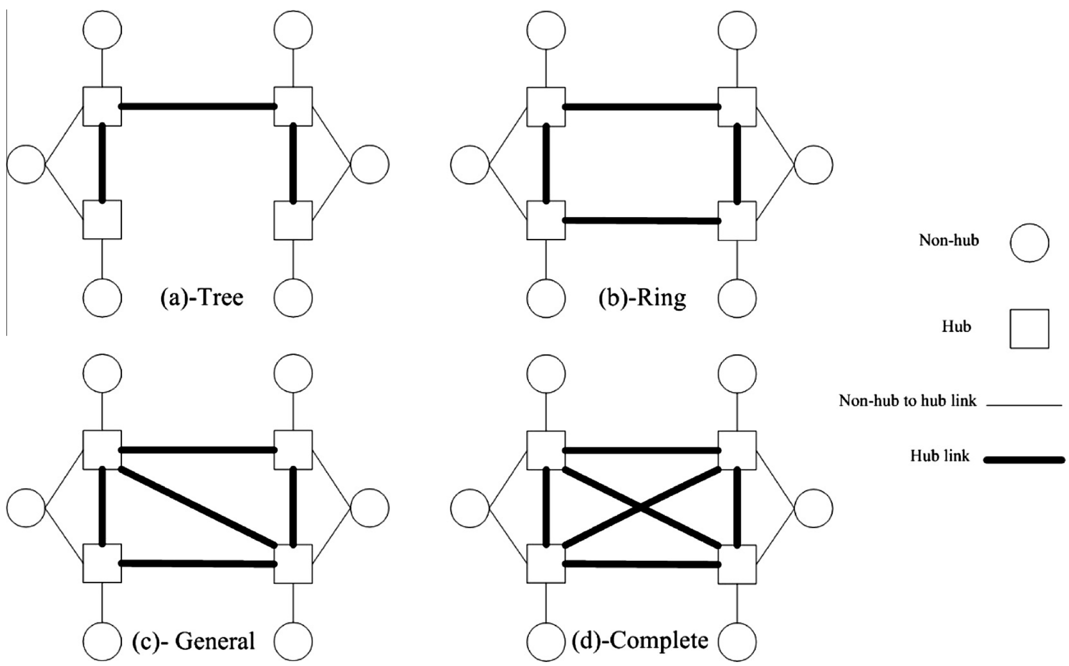


**Fig. 1.** Examples of the hub networks.