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A robust counterpart approach to the bi-objective emergency medical service design problem

Zhi-Hai Zhang *, Hai Jiang

Department of Industrial Engineering, Tsinghua University, Beijing 100084, China

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ABSTRACT

The paper presents a bi-objective robust program to design a cost-responsiveness efficient emergency medical services (EMS) system under uncertainty. The proposed model simultaneously determines the location of EMS stations, the assignment of demand areas to EMS stations, and the number of EMS vehicles at each station to balance cost and responsiveness. We develop a robust counterpart approach to cope with the uncertain parameters in the EMS system. Extensive numerical studies are performed to demonstrate the benefits of our robust optimization approach.

1. Introduction

Emergency Medical Services (EMS) are designed to provide people with sudden life-threatening emergencies with quick, effective medical care. Extensive research has been conducted to study the location and sizing of emergency medical services. The objectives considered include costs minimization, coverage equity maximization, area coverage maximization, call coverage maximization, and so on [1]. In recent years, the multi-objective EMS design problem has attracted a lot of attention. Literature on the deterministic multi-objective design problem is abundant. A sampling of these research includes [2–6].

To capture the uncertainties in the system, Harewood [7] embeds a queuing model into a bi-objective Maximum Availability Location Problem for an EMS system. The two objectives considered are the maximization of the serviced population and the minimization of the cost of covering the population. Later, Araz et al. [8] propose a fuzzy multi-objective maximal covering location model to determine the best base locations for a limited number of vehicles so that service level objectives are optimized. Three objectives are considered: maximization of the population covered by one vehicle, maximization of the population with backup coverage, and minimization of the total travel distance. More recently, Bozorgi-Amiri et al. [9] present a multi-objective robust stochastic programming approach for disaster relief logistics under uncertainty by using scenarios approach. The model attempts to minimize the sum of the mean and the variance of the total cost of the relief chain; meanwhile, it maximizes the affected areas' satisfaction levels through minimizing the sum of the maximum shortage in the affected areas. Chanta et al. [10] study an emergency vehicle location problem with the objectives of maximization of the number of requested calls within a required response time limit, and the reduction of disparity in service between rural and urban citizens. A well-known hypercube model is embedded into the model to calculate the expected number of emergency calls.

The above research on the stochastic multi-objective EMS design problem is limited in that: (a) The queuing models [10] are often too complex to tackle large-scale design problems; (b) The linearization of the chance constraints is not an easy

* Corresponding author. Tel.: +86 010 62772874; fax: +86 010 62794399.
E-mail addresses: zhzhang@tsinghua.edu.cn (Z.-H. Zhang), haijiang@tsinghua.edu.cn (H. Jiang).

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task [11]; and (c) The associated uncertainties in the EMS system is difficult, if not impossible, to quantify through probability distribution functions due to the scarcity of data. To address these challenges, we develop a robust counterpart (RC) approach [12] to the bi-objective emergency medical service design problem under uncertainty. In the RC approach, the uncertainty is not described by a probability density function or scenarios. Rather, it is "deterministic" and known to belong to an uncertainty set [12,13]. This set defines the limits on uncertainty that a solution will be immunized against. Hence, instead of immunizing the solution in a probabilistic sense, the decision-maker searches for a solution that is optimal for all possible realizations of the uncertainty set.

The main contributions of this paper are summarized as follows:

- To the best of our knowledge, we are the first to develop a robust counterpart approach to solve the bi-objective EMS design problem, in which the associated uncertainties are hard to quantify through probability distribution functions due to the scarcity of data; and
- Our model is capable of finding Pareto-optimal solutions for costs and response times by optimizing the location of the EMS stations and the number of emergency vehicles at each station.

The remainder of this paper is organized as follows. A short introduction of the RC approach is presented in Section 2. Section 3 provides a deterministic bi-objective program, while Section 4 presents its robust counterpart. Extensive numerical studies are carried out in Section 5. Finally, Section 6 concludes this paper and outlines possible future research directions.

2. A brief introduction on the RC approach

Robustness analysis takes into account uncertainty or imprecision in model parameters so as to produce decisions that are more robust, that is, behave reasonably well under uncertainty. The robust counterpart approach [12] is a popular means to increase the robustness of a model. We present a brief introduction on this approach in this section.

Consider the following problem subject to uncertain coefficients:

$$\begin{aligned}(LP^D)\min \quad & C^T X,\\ \text{s.t.} \quad & a_i^T X \leqslant b_i, \quad \forall i,\\ & X \in \mathbb{R}^n.\end{aligned} \tag{1}$$

where $a_i = [a_{i1}, a_{i2}, \ldots, a_{in}]^T$, $b_i \in \mathbb{R}$. In a typical deterministic model, we assume the exact values of C, a_i and b_i are known. The RC approach considers uncertain parameters in the model. Without loss of generality, we assume the uncertainty affect vector a_i. According to [12], the RC of constraint (1) is derived by addressing the following mathematical programs:

$$\max_{a_i \in \mathbb{U}} a_i^T X \leqslant b_i, \quad \forall i.$$

where $\mathbb{U}$ represents an uncertainty set for a_i. In this way, the solution is robust under uncertainty if the maximum value of the left-hand side of constraint (1) is still less than the right-hand side of the constraint.

To formulate the uncertainty of a_i, two types of uncertainty sets are common in literature: box and ellipsoidal uncertainty sets. Let a_{ij} denote an uncertain entry in the vector. The box uncertainty set is defined as a box of form $\mathbb{U}^B = \{a_{ij} \in \mathbb{R} : |a_{ij} - \bar{a}_{ij}| \leqslant \epsilon H_{ij}\}$, where $\bar{a}_{ij}$ is the nominal value of a_{ij}, H_{ij} represents the uncertainty scale for this given entry, and ϵ is the uncertainty level common across all the entries [12]. The RC with box uncertainty set often leads to over-conservative solutions because the worst cases of every uncertain parameter are satisfied simultaneously. To overcome this drawback, the ellipsoidal uncertainty set is introduced to reflect the fact that the coefficients of the constraints are not expected to be simultaneously at their worst values. The ellipsoidal uncertainty set is defined as:

$$\mathbb{U}^E = \left\{a_i \in \mathbb{R}^n : (a_i - \bar{a}_i)^T \Sigma^{-1} (a_i - \bar{a}_i) \leqslant \Omega^2\right\}.$$

where $\bar{a}_i$ is the vector of a_i's nominal values, Σ is a positive definite matrix, and Ω is a safety parameter indicating the amount of uncertainty. By using an affine transformation, it can also be expressed as a ball of radius Ω:

$$\mathbb{U}^E = \{a_i \in \mathbb{R}^n : a_i = \bar{a}_i + \Delta\xi, \|\xi\| \leqslant \Omega\},$$

where $\Delta = \Sigma^{\frac{1}{2}}$.

Under ellipsoidal uncertainty, the RC of Constraint (1) can be derived as follows:

$$\begin{aligned}&\max_{a_i \in \mathbb{U}^E} a_i^T X \leqslant b_i,\\ &\iff \max_{\|\xi\| \leqslant \Omega} (\bar{a}_i + \Delta\xi)^T X \leqslant b_i,\\ &\iff \bar{a}_i^T X + \Omega\|\Delta X\| \leqslant b_i.\end{aligned}$$

The resulting RC is a conic quadratic constraint [14] which can be solved efficiently by interior-point methods. The major advantages of the RC approach are: (1) We are not required to have the probability information about the uncertain param-