Applied Mathematical Modelling 38 (2014) 1159–1169

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Vibration of nanobeams of different boundary conditions with multiple cracks based on nonlocal elasticity theory

Hossein Roostai *, Mohammad Haghpanahi

Department of Mechanical Engineering, Iran University of Science & Technology (IUST), Tehran, Islamic Republic of Iran

ARTICLE INFO

Article history:
Received 4 May 2012
Received in revised form 27 September 2012
Accepted 5 August 2013
Available online 20 August 2013

Keywords:
Nonlocal elasticity theory
Vibration
Crack
Nanobeam

ABSTRACT

The aim of this paper is to study the free vibration of nanobeams with multiple cracks. The analysis procedure is based on nonlocal elasticity theory. This theory states that stress at a point is a function of strains at all points in the continuum. The nonlocal elasticity theory becomes significant for small length scale in micro and nanostructures. The effects of nonlocality, crack location and crack parameter are investigated on the natural frequencies of the cracked nanobeam. In this study, analytical solutions are given for cracked Euler–Bernoulli nanobeams of different boundary conditions.

1. Introduction

In recent years, researchers are very interested to investigate about micro and nanostructures. Nanoscale beams are commonly used in nanoelectromechanical systems (NEMS) devices (e.g., resonators) with different boundary and loading conditions [1].

The continuum elasticity theory is not sufficient to analyze the small scale structures and for a better prediction, the nonlocal elasticity theory can be used. This continuum elasticity theory assumes that stress at a point is a function of strain at that point, whereas in the nonlocal elasticity theory, stress at a point is a function of strains at all points in the continuum. The nonlocal elasticity theory has been used to investigate wave propagation, dislocation mechanics, crack problems and elastic waves [2–5]. Furthermore, some other works are cited herein. Wang and Varadan [6] developed a nonlocal continuum mechanics model to study the vibration of both single-walled nanotubes and double-walled nanotubes via elastic beam theories. Murmu and Pradhan [7] proposed a thermal vibration model to analyze the natural frequency of single-walled carbon nanotubes using the nonlocal elasticity theory. Aydogdu [8] used a compact generalized beam theory to analyze bending, buckling, and vibration of nanoscale beams using local and nonlocal elasticity. Niu et al. [9] presented formulations of nonlocal third-order nanobeam theories for bending, buckling and vibration of nanobeams via the variational principle. Chowdhury et al. [10] focused on the effect of the heterogeneous end constraints on the low frequency vibration modes of multiwall carbon nanotubes. Recently, Chowdhury et al. [11] considered the vibrational properties of zigzag and armchair single wall carbon nanotubes using the molecular mechanics approach. Phadikar and Pradhan [12] established variational formulation and finite element analysis for nonlocal elastic nanobeams and nanoplates. More recently, Murmu and Adhikari [13,14] have investigated nonlocal effects in the longitudinal and transverse vibration of double nanorod and nanobeam systems.

* Corresponding author. Tel.: +98 9183514539.
E-mail addresses: hossein.roostai@gmail.com (H. Roostai), mhaghpanahi@iust.ac.ir (M. Haghpanahi).

0307-904X/$ - see front matter
http://dx.doi.org/10.1016/j.apm.2013.08.011

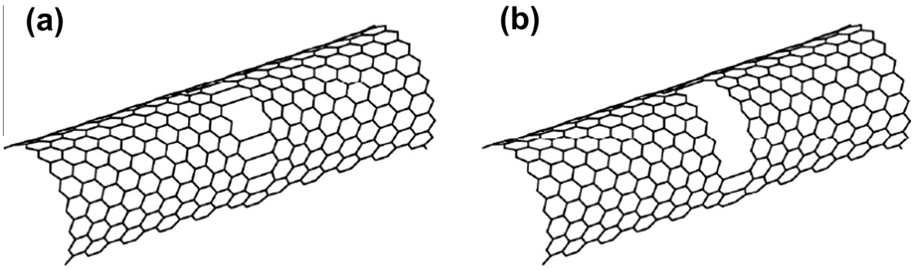

Fig. 1. Evolution of the crack in the nanotube [16].

A crack can be introduced as a local change in a structure's stiffness. Two types of initial defects in nanotubes have been considered by authors. Type I is initial defect comprising of 10% weakening of one bond and Type II is creating a vacancy in nanotubes [15]. Belytschko et al. [16] studied the fracture of carbon nanotubes by molecular mechanics and dynamics. They presented an evolution of the crack in the nanotube such that the bond-breaking spreads sideways after the initially weakened bond fails (Fig. 1). Crack in a single-walled carbon nanotube is oriented in the lateral direction [15]. In fact, one important aspect of the cracking phenomena is the appearance of the first few broken bonds [17]. Recently, an extended finite element method has been used to simulate crack propagation in carbon nanotubes by Joshi et al. [15].

Recently, Loya et al. [18] modified the classical cracked Euler–Bernoulli beam model, based on the nonlocal elasticity theory to study the free vibration of cracked nanobeams. They divided the cracked element into two segments connected by a rotational spring located at the cracked section. In this paper, free vibration of nanobeams with multiple cracks based on nonlocal elasticity theory has been reported. Analytical solutions are given for cracked Euler–Bernoulli nanobeams of different boundary conditions. Cracks are modeled as rotational springs following the classical approach proposed by Loya et al. [18].

2. Analysis

2.1. Free vibration of an intact nanobeam

A schematic diagram of nanobeam with multiple cracks subjected to axial force is depicted in Fig. 2. The nanobeam has n cracks located at $X_i(i = 1, \ldots, n)$.

Nonlocal constitutive relations for present nanobeam can be written as follows:

$$\left(1 - (e_0 a)^2 \nabla^2\right)\sigma_{ij} = \sigma^c_{ij}, \tag{1}$$

where σ_{ij}, σ^c_{ij} and ∇^2 are the stress tensor of the nonlocal elasticity, the classical local stress tensor and the Laplace operator, respectively. $e_0 a$ is a nanolength scale where a is an internal characteristic length (e.g. C–C bond length, granular distance, lattice parameter) and e_0 is a calibration constant appropriate to each material. The magnitude of e_0 is determined experimentally or estimated such that the relations of the nonlocal elasticity model could provide satisfactory approximations to the atomic dispersion curves of the plane waves with those obtained from the atomistic lattice dynamics [13]. A conservative estimate of the scale coefficient $e_0 a < 2.0$ nm for a single-walled carbon nanotube has been proposed by Wang [4]. The

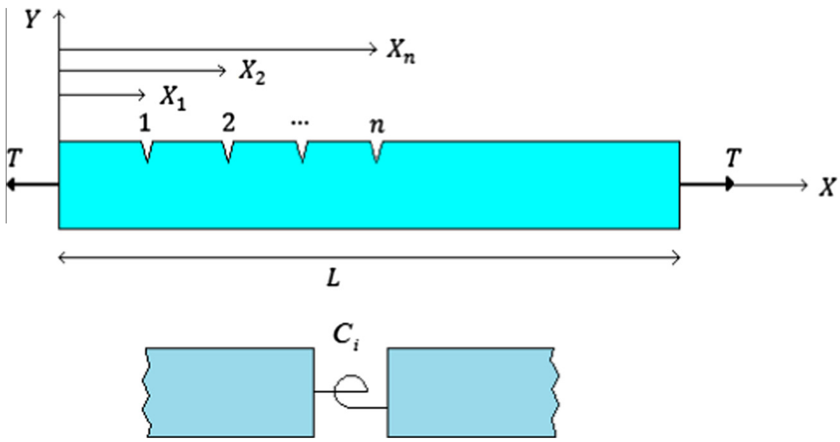

Fig. 2. Nanobeam with multiple cracks subjected to axial force.