Applied Mathematical Modelling 38 (2014) 1170–1175

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Short communication

Single valued neutrosophic cross-entropy for multicriteria decision making problems

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ARTICLE INFO

Article history:
Received 19 October 2011
Received in revised form 8 March 2013
Accepted 4 July 2013
Available online 23 July 2013

Keywords:
Cross entropy
Single valued neutrosophic set
Single valued neutrosophic cross entropy
Decision-making

ABSTRACT

A single valued neutrosophic set (SVNS) is an instance of a neutrosophic set, which give us an additional possibility to represent uncertainty, imprecise, incomplete, and inconsistent information which exist in real world. It would be more suitable to apply indeterminate information and inconsistent information measures. In this paper, the cross entropy of SVNSs, called single valued neutrosophic cross entropy, is proposed as an extension of the cross entropy of fuzzy sets. Then, a multicriteria decision-making method based on the proposed single valued neutrosophic cross entropy is established in which criteria values for alternatives are SVNSs. In decision making process, we utilize the single-valued neutrosophic weighted cross entropy between the ideal alternative and an alternative to rank the alternatives corresponding to the cross entropy values and to select the most desirable one(s). Finally, a practical example of the choosing problem of suppliers is provided to illustrate the application of the developed approach.

1. Introduction

Entropy is very important for measuring uncertain information. The fuzzy entropy was first introduced by Zadeh [1,2]. The starting point for the cross entropy approach is information theory as developed by Shannon [3]. Kullback–Leibler [4] proposed a measure of the "cross entropy distance" between two probability distributions. Later, Lin [5] proposed a modified cross-entropy measure. Shang and Jiang [6] proposed a fuzzy cross entropy measure and a symmetric discrimination information measure between fuzzy sets. As an intuitionistic fuzzy set is a generalization of a fuzzy set, Vlachos and Sergiadis [7] proposed an intuitionistic fuzzy cross-entropy based on an extension of the De Luca-Termini nonprobabilistic entropy [8] and applied it to the pattern recognition, medical diagnosis, and image segmentation. Then, Zhang and Jiang [9] defined a vague cross-entropy between vague sets (VSs) by analogy with the cross entropy of probability distributions and applied it to the pattern recognition and medical diagnosis, and then Ye [10] has investigated the fault diagnosis problem of turbine according to the cross entropy of vague sets. Furthermore, Ye [11] has applied the intuitionistic fuzzy cross entropy to multicriteria fuzzy decision-making problems. Ye [12] proposed an interval-valued intuitionistic fuzzy cross-entropy based on the generalization of the vague cross-entropy [9] and applied it to multicriteria decision-making problems.

Smarandache [13] originally introduced neutrosophy. It is a branch of philosophy which studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra. Neutrosophic set is a powerful general formal framework, which generalizes the concept of the classic set, fuzzy se, interval valued fuzzy set, intuitionistic fuzzy set, interval-valued intuitionistic fuzzy set, paraconsistent set, dialetheist set, paradoxist set, and tautological set [13]. In the neutrosophic set, indeterminacy is quantified explicitly and truth-membership, indeterminacy-membership, and

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http://dx.doi.org/10.1016/j.apm.2013.07.020

falsity-membership are independent. This assumption is very important in many applications such as information fusion in which the data are combined from different sensors. Recently, neutrosophic sets had mainly been applied to image processing [14,15] in engineering field.

Intuitionistic fuzzy sets and interval valued intuitionistic fuzzy sets can only handle incomplete information but not the indeterminate information and inconsistent information which exist commonly in real situations. For example, when we ask the opinion of an expert about certain statement, he or she may that the possibility that the statement is true is between 0.5 and 0.7, and the statement is false is between 0.2 and 0.4, and the degree that he or she is not sure is between 0.1 and 0.3. For neutrosophic notation, it can be expressed as $x([0.5,0.7], [0.1,0.3], [0.2,0.4])$. Here is another example, suppose there are 10 voters during a voting process. In time t_1, four vote "yes", three vote "no" and three are undecided. For neutrosophic notation, it can be expressed as $x(0.4,0.3,0.3)$; in time t_2, two vote "yes", three vote "no", two give up, and three are undecided, then it can be expressed as $x(0.2,0.3,0.3)$. The mentioned information is beyond the scope of the intuitionistic fuzzy set. So the notion of neutrosophic set is more general and overcomes the aforementioned issues.

The neutrosophic set generalizes the above mentioned sets from philosophical point of view. From scientific or engineering point of view, the neutrosophic set and set-theoretic operators need to be specified. Otherwise, it will be difficult to apply in the real applications. Therefore, Wang et al. [16] proposed a single valued neutrosophic set (SVNS) and provide the set-theoretic operators and various properties of SVNSs. Recently, Ye [17] proposed Similarity measures between interval neutrosophic sets and applied them to multicriteria decision-making problems under the interval neutrosophic environment.

On one hand, a SVNS is an instance of a neutrosophic set, which give us an additional possibility to represent uncertainty, imprecise, incomplete, and inconsistent information which exist in real world. It would be more suitable to apply indeterminate information and inconsistent information measures in decision-making. However, the connector in the fuzzy set is defined with respect to T, i.e. membership only, hence the information of indeterminacy and nonmembership is lost. The connectors in the intuitionistic fuzzy set are defined with respect to T and F, i.e. membership and nonmembership only, hence the indeterminacy is what is left from 1. While in the SVNS, they can be defined with respect to any of them (no restriction). So the notion of SVNSs is more general and overcomes the aforementioned issues. On the other hand, SVNSs can be used for the scientific and engineering applications because SVNS theory is valuable in modeling uncertain, imprecision and inconsistent information. Due to its ability to easily reflect the ambiguous nature of subjective judgments, the SVNS is suitable for capturing imprecise, uncertain, and inconsistent information in the multicriteria decision-making analysis. However, to the best of our knowledge, there is no work addressing the multicriteria decision-making problems in single valued neutrosophic setting. Therefore, the main purposes of this paper were (1) to present the cross entropy of SVNSs, called single valued neutrosophic cross-entropy, and (2) to establish a multicriteria decision-making method by use of the cross entropy of SVNSs. In the decision-making process, we utilize the single-valued neutrosophic weighted cross entropy between the ideal alternative and an alternative to rank the alternatives corresponding to the cross entropy values, and to select the most desirable one(s).

The rest of paper is organized as follows. In Section 2, we introduce the some concepts of neutrosophic sets and SVNSs, and some operators for SVNSs. In Section 3, a cross-entropy measure between SVNSs (called single valued neutrosophic cross-entropy) is proposed as an extension of the fuzzy cross entropy measure. Section 4 establishes a multicriteria decision-making method based on the proposed cross-entropy of SVNSs. In Section 5, a practical example of the choosing problem of suppliers is given to illustrate the application of the developed approach. Finally, some final remarks and further work are given in Section 6.

2. Some concepts of neutrosophic sets and SVNSs

2.1. Neutrosophic sets

Neutrosophic set is a part of neutrosophy, which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra [13], and is a powerful general formal framework, which generalizes the above mentioned sets from philosophical point of view.

Smarandache [13] gave the following definition of a neutrosophic set.

Definition 1 [13]. Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, a indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0^-,1^+[$. That is $T_A(x)$: $X \to]0^-,1^+[$, $I_A(x)$: $X \to]0^-,1^+[$, and $F_A(x)$: $X \to]0^-,1^+[$.

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leqslant \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leqslant 3^+$.

Definition 2 [13]. The complement of a neutrosophic set A is denoted by A^c and is defined as $T_A{}^c(x) = \{1^+\} \ominus T_A(x)$, $I_A{}^c(x) = \{1^+\} \ominus I_A(x)$, and $F_A{}^c(x) = \{1^+\} \ominus F_A(x)$ for every x in X.

Definition 3 [13]. A neutrosophic set A is contained in the other neutrosophic set B, $A \subseteq B$ if and only if $\inf T_A(x) \leqslant \inf T_B(x)$, $\sup T_A(x) \leqslant \sup T_B(x)$, $\inf I_A(x) \geqslant \inf I_B(x)$, $\sup I_A(x) \geqslant \sup I_B(x)$, $\inf F_A(x) \geqslant \inf F_B(x)$, and $\sup F_A(x) \geqslant \sup F_B(x)$ for every x in X.