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Short communication

Analysis of large amplitude free vibrations of clamped tapered beams on a nonlinear elastic foundation

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ABSTRACT

The purpose of this paper is to present efficient and accurate analytical expressions for large amplitude free vibration analysis of single and double tapered beams on elastic foundation. Geometric nonlinearity is considered using the condition of inextensibility of neutral axis. Moreover, the elastic foundation consists of a linear and cubic nonlinear parts together with a shearing layer. The nonlinear governing equation is solved by employing the variational iteration method (VIM). This study shows that the second-order approximation of the VIM leads to highly accurate solutions which are valid for a wide range of vibration amplitudes. The effects of different parameters on the nonlinear natural frequency of the beams are studied under different mode shapes. The results of the present work are also compared with those available in the literature and a good agreement is observed.

1. Introduction

Non-prismatic beams, i.e. beams their cross-section varies gradually or abruptly along the length play an important role in different fields of engineering [1]. They can be used in optimization of weight and strength of the structures as well as in architectural and aesthetical aspects of structural engineering design [2]. These particular characteristics enable the engineers to design and construct fine and precise structures, i.e. aerospace structures.

In addition to the earlier efforts attempted to find the linear response (linear natural frequencies and mode shapes) of tapered beams (see e.g. [3–5] among others), nonlinear vibrations of tapered beams have attracted a great deal of interest [6–8]. Here we present a brief review of the analytical and numerical investigations performed regarding modeling of tapered beams especially within large amplitude vibrations. Rao and Rao [9] presented a simple formulation for the problem of the large amplitude free vibrations of tapered beams. Applying an iterative numerical scheme, they obtained the solution of this problem. The analysis of an elastically restrained tapered cantilever beam using the harmonic balance and the time transformation methods is performed by Abdel-Jaber et al. [10]. The nonlinear flexural vibration analysis of tapered Timoshenko beams is conducted numerically by Liao and Zhong [11]. Free vibration and stability analysis of axially functionally graded tapered Timoshenko beams are studied through a finite element approach by Shahba et al. [12]. There are also several numerical investigations about different aspects of vibration of tapered beams. In this regard, one may refer to [13–15].

An exact solution for the post-buckling of a symmetrically laminated composite beam with different boundary conditions is presented in [16]. Malekzadeh and Vosoughi [17] have studied the large amplitude free vibration analyses of symmetric

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angle-ply laminated thin beams on nonlinear elastic foundation with elastically restrained against rotation edges. Patel et al. [18] have investigated nonlinear free flexural vibrations and post-buckling of laminated orthotropic beams resting on a class of two parameter elastic foundation employing a three-nodded shear flexible beam element.

Although in the analytical solution of nonlinear equations, we encounter some difficulties and the application of different numerical techniques seems to be necessary, but closed form solutions even if they are approximate solutions are more interesting. This is due to various advantages of analytical methods such as ease of parametric studies and consideration of the physics of the problem. Generally in a given nonlinear problem, it is often difficult to find an analytical solution unless a number of simplifying assumptions are considered. Otherwise, application of different numerical techniques is unavoidable. However, it is normally hard to have a complete and indispensable understanding of a nonlinear problem out of numerical results. In addition, numerical difficulties appear if a nonlinear problem has singularities or multiple solutions.

Among several analytical methods, Variational Iteration Method (VIM) is one of the most accurate and efficient methods of studying vibrations of nonlinear systems ([19–23]). In our previous paper ([24]), this method was used to analyze the large amplitude free vibration and post-buckling of unsymmetrically laminated composite beams on elastic foundation and the accuracy of the method was investigated.

The paper is organised as follows. In Section 2, the mathematical model of the tapered beams is constructed based on the Lagrange method. Applying the assumed mode shape method, the governing equation of motion will be reduced. Incorporating the inextensibility condition in the model, a kinematic dependency between axial and transverse displacements will be produced. Moreover, effect of nonlinear elastic foundation is considered in calculating the Lagrangian. Using variational iteration method, the reduced equation of motion is solved in Section 3. Analytical expressions for the nonlinear fundamental natural frequency and deflection of both single and double tapered beams are obtained using the VIM. The results are given in Section 4 and also a parametric study (material and geometrical parameters) is performed in this section. Results are compared with those available from the literature and also with the results of numerical study. Finally, we present a summary and draw conclusions in Section 5.

2. Model development

Assuming the Euler–Bernoulli beam theory, the strain energy U and the kinetic energy K for a tapered beam which has been shown schematically in Fig. 1, are given as:

$$\begin{aligned} U &= \frac{1}{2}\int_0^1 EI(\xi)\kappa^2 d\xi, \\ K &= \frac{1}{2}\int_0^1 \rho l A(\xi)\left(\dot{u}^2+\dot{w}^2\right)d\xi, \end{aligned} \tag{1}$$

where $\xi = s/l$ and κ is the curvature of the beam neutral axis. ζ is a dimensionless parameter which varies from ε^* (according to the free end) to 1 (according to the clamped end). Also u and w are axial and transverse displacements while s, A and I represent length parameter, cross section area and moment of inertia, respectively. Moreover, mechanical properties of the material are density and elastic modulus which are denoted by ρ and E, respectively. Applying the change of parameter $w' = l\sin\varphi$ (it is noted that prime represents the derivative with respect to ξ) and using the fact that $\kappa = \varphi'/l$, after some manipulations, it is easily concluded that:

$$\varphi' = (1-(w'/l)^2)^{\frac{1}{2}} w''/l. \tag{2}$$

Applying the taylor series to (2), we may obtain:

$$\varphi' = w''/l(1+\frac{1}{2}(w'/l)^2). \tag{3}$$

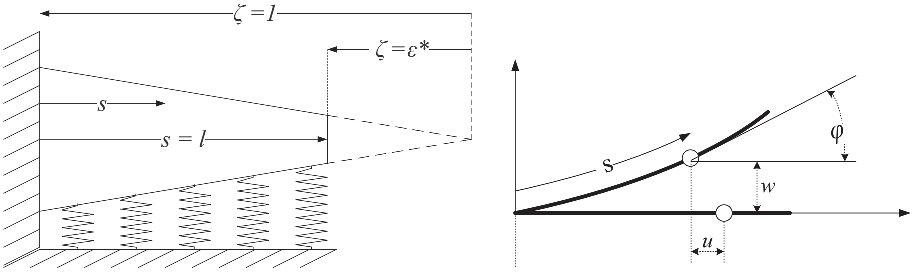

Fig. 1. Initial configuration of the tapered beam on elastic foundation (left). Deflection of the neutral axis (right).