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Auxiliary model based least squares identification method for a state space model with a unit time-delay ☆

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ABSTRACT

This paper considers parameter estimation problems for state space systems with time-delay. By means of the property of the shift operator, the state space systems are transformed into the input–output representations and an auxiliary model identification method is presented to estimate the system parameters. Finally, an example is provided to test the effectiveness of the proposed algorithm.

1. Introduction

State space models have wide applications in many areas, e.g., system modeling and control [1–5], system identification [6–11], signal processing [12–15], adaptive filtering [16–18]. There exist many parameter estimation methods for system modeling, such as the maximum likelihood least squares methods [19–21], the auxiliary model identification methods [22–26], the stochastic gradient methods [27–29], the iterative estimation algorithms [30–35] and other identification methods [36–39]. In this literature, Dehghani and Nikravesh studied the nonlinear state space model identification of the synchronous generators [40]; Schön et al. discussed the system identification of the nonlinear state-space models [41]; Du et al. studied the indirect identification of continuous-time delay systems from step responses [42].

Most contributions for systems with time-delays in the literature focus on the studies of control schemes. For example, Yan and Shi discussed the robust discrete-time sliding mode control for the uncertain systems with time-varying state delay [43]; Shi and Yu studied the output feedback stabilization of the networked control systems with random delays modeled by Markov chains [44]. This paper considers identification problems of time-delay control systems based on the auxiliary model identification idea. The auxiliary model method is a new-type parameter estimation one and can deal with identification problems with the information vector including unknown internal variables [11,26]. In this literature, Ding and Chen presented an auxiliary model based recursive least squares algorithm for dual-rate sampled-data systems [45]; Chen et al. presented the auxiliary model based multi-innovation algorithms for multivariable nonlinear systems [46]; Han et al. discussed an auxiliary model identification method for multirate multi-input systems based on least squares [47] and Han and Ding

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presented a multi-innovation stochastic gradient identification algorithm for multirate multi-input systems using the multi-innovation identification theory [48].

This paper presents an auxiliary model based identification algorithm of the input–output representations corresponding to state space systems with time-delays. The basic idea is, by means of the property of the shift operator, to transform the state space model with a time-delay into an input–output representation and then to identify the parameters of the input–output representation based on the auxiliary model identification technique. The proposed method has the advantage of handling the unmeasured variables in the information vector.

Briefly, this paper is organized as follows. Section 2 derives an input–output representation related to the state space model with time-delay. Section 3 presents an auxiliary model based recursive least squares algorithm. Section 4 provides an illustrative example to verify the effectiveness of the proposed algorithm. Finally, concluding remarks are given in Section 5.

2. The input–output representation

Let us define some notations. $\hat{\boldsymbol{\theta}}(t)$ represents the estimate of $\boldsymbol{\theta}$ at time t; "$\boldsymbol{A} =: \boldsymbol{X}$" or "$\boldsymbol{X} := \boldsymbol{A}$" stands for "$\boldsymbol{A}$ is defined as $\boldsymbol{X}$"; the symbol $\boldsymbol{I}$ ($\boldsymbol{I}_n$) stands for an identity matrix of appropriate sizes ($n \times n$); z represents a unit forward shift operator: $z\boldsymbol{x}(t) = \boldsymbol{x}(t+1)$ and $z^{-1}\boldsymbol{x}(t) = \boldsymbol{x}(t-1)$; the superscript T denotes the matrix/vector transpose; 1_n represents an n-dimensional column vector whose elements are all unity; adj[$\boldsymbol{X}$] denotes the adjoint matrix of the square matrix $\boldsymbol{X}$: $\mathrm{adj}[\boldsymbol{X}] = \det[\boldsymbol{X}]\boldsymbol{X}^{-1}$; det[$\boldsymbol{X}$] denotes the determinant of the square matrix $\boldsymbol{X}$.

Consider the following state space system with a unit time-delay,

$$\boldsymbol{x}(t+1) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{x}(t-1) + \boldsymbol{f}u(t), \tag{1}$$

$$y(t) = \boldsymbol{c}\boldsymbol{x}(t) + du(t) + v(t), \tag{2}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}$ is the system input, $y(t) \in \mathbb{R}$ is the system output, $v(t) \in \mathbb{R}$ is a random noise with zero mean, $\boldsymbol{A} \in \mathbb{R}^{n\times n}$, $\boldsymbol{B} \in \mathbb{R}^{n\times n}$, $\boldsymbol{f} \in \mathbb{R}^n$, $\boldsymbol{c} \in \mathbb{R}^{1\times n}$ and $d \in \mathbb{R}$ are the system parameter matrices/vectors.

Since Eq. (1) contains the term $\boldsymbol{x}(t-1)$, we say that this state equation has a unit time-delay. If we change $\boldsymbol{x}(t-1)$ in (1) into $\boldsymbol{x}(t-d)$, it is a d-time-delay (d is an integer).

The following transforms the time-delay state space model in (1) and (2) into an input–output representation and gives its identification model.

Lemma 1. *For the state space model in (1) and (2), the transfer function from the input $u(t)$ to the output $y(t)$ is given by*

$$G(z) := \boldsymbol{c}(z^2\boldsymbol{I} - \boldsymbol{A}z - \boldsymbol{B})^{-1}\boldsymbol{f}z + d = \frac{\boldsymbol{c}\,\mathrm{adj}[z^2\boldsymbol{I} - \boldsymbol{A}z - \boldsymbol{B}]\boldsymbol{f}z + \det[z^2\boldsymbol{I} - \boldsymbol{A}z - \boldsymbol{B}]d}{\det[z^2\boldsymbol{I} - \boldsymbol{A}z - \boldsymbol{B}]}.$$

Proof. Using the properties of the shift operator z, Eq. (1) can be rewritten as

$$z\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{x}(t) + z^{-1}\boldsymbol{B}\boldsymbol{x}(t) + \boldsymbol{f}u(t).$$

Multiplying both sides by z gives

$$z^2\boldsymbol{x}(t) = \boldsymbol{A}z\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{x}(t) + \boldsymbol{f}zu(t)$$

or

$$\boldsymbol{x}(t) = (z^2\boldsymbol{I} - \boldsymbol{A}z - \boldsymbol{B})^{-1}\boldsymbol{f}zu(t).$$

Substituting the above $x(t)$ into (2) gives the output equation:

$$y(t) = \boldsymbol{c}(z^2\boldsymbol{I} - \boldsymbol{A}z - \boldsymbol{B})^{-1}\boldsymbol{f}zu(t) + du(t) + v(t) = [\boldsymbol{c}(z^2\boldsymbol{I} - \boldsymbol{A}z - \boldsymbol{B})^{-1}\boldsymbol{f}z + d]u(t) + v(t) = G(z)u(t) + v(t). \tag{3}$$

Then we have the transfer function of the system from the input $u(t)$ to the output $y(t)$:

$$G(z) = \boldsymbol{c}(z^2\boldsymbol{I} - \boldsymbol{A}z - \boldsymbol{B})^{-1}\boldsymbol{f}z + d =: \frac{\beta(z)}{\alpha(z)}, \tag{4}$$

where $\alpha(z)$ is the denominator of the transfer function, i.e., the characteristic polynomial of the system, and $\beta(z)$ is the numerator of the transfer function, and they are defined by

$$\alpha(z) := z^{-2n}\det[z^2\boldsymbol{I} - \boldsymbol{A}z - \boldsymbol{B}] = z^{-2n}(z^{2n} + \alpha_1 z^{2n-1} + \alpha_2 z^{2n-2} + \cdots + \alpha_{2n}) = 1 + \alpha_1 z^{-1} + \alpha_2 z^{-2} + \cdots + \alpha_{2n}z^{-2n}, \tag{5}$$

$$\beta(z) := z^{-2n}\boldsymbol{c}\,\mathrm{adj}[z^2\boldsymbol{I} - \boldsymbol{A}z - \boldsymbol{B}]\boldsymbol{f}z + \alpha(z)d = \beta_0 + \beta_1 z^{-1} + \beta_2 z^{-2} + \cdots + \beta_{2n}z^{-2n}. \tag{6}$$

Substituting (4) into (3) gives the input–output representation of the time-delay state space model in (1) and (2), i.e., an output error model: