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Two-dimensional thermoelastic analysis of beams with variable thickness subjected to thermo-mechanical loads

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ABSTRACT

Two-dimensional thermoelastic analysis for simply supported beams with variable thickness and subjected to thermo-mechanical loads is investigated. An approximate analytical method is proposed. Firstly, the heat conduction equation is analytically solved to obtain the temperature distributions for two kinds of boundary conditions at the beam ends, which are the harmonic series with unknown coefficients. Then the two-dimensional equilibrium differential equations are analytically solved to obtain the displacement component series with unknown coefficients and the stress component series is obtained. The unknown coefficients in the temperature series and the stress component series are approximately determined by using the upper surface and lower surface conditions of the beam. With the proposed procedure, the solutions satisfy the governing differential equations, the loading conditions, and the simply supported end conditions. The proposed solution method shows a good convergence and the results agree well with those obtained from the commercial finite element software ANSYS. Several examples are used to demonstrate the effectiveness of the proposed solution method. The simultaneous effects of temperature change and applied mechanical load on the behavior of the beam are examined.
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1. Introduction

It is well known that non-uniform beams may provide a better or more suitable distribution of mass and strength than uniform beams and therefore can meet special functional requirements in architecture, robotics, aeronautics and other innovative engineering applications. A great deal of attention had been paid to non-uniform beams and most of them were based on the Bernoulli–Euler beam theory and the Timoshenko beam theory. For example, Lee et al. [1] studied the static deflection of a non-uniform Bernoulli–Euler beam with general elastic end restraints. Romano and Zingone [2] obtained the closed form solutions of beams with varying rectangular cross-section by solving the fourth-order differential equations with variable coefficients. Zenkour [3] studied the elastic behavior of an orthotropic beam/one-dimensional plate of uniform and variable thickness. Lee and Kuo [4] made the static analysis of non-uniform Timoshenko beams. Romano [5] obtained the closed form solutions for Timoshenko beams with linearly and parabolically varying depth. Sapountzakis and Panagos [6] presented the non-linear analysis of a composite Timoshenko beam with generally variable cross-section undergoing moderate large deflections by employing the analog equation method.

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It should be mentioned that the Bernoulli–Euler beam theory for thin isotropic beams and the Timoshenko beam theory for moderately thick beams generally can give accurate solutions with negligible error. However, with the increasing of the thickness, the transverse shear deformation plays a more important role. In such a case, the classical beam theory based on the Euler–Bernoulli hypothesis will lead to inaccurate or even erroneous results. Although the Timoshenko beam theory takes into account the effects of transverse shear deformation and rotatory inertia, it is difficult to determine the required shear correction factors for various thick beams. In such a case, the two-dimensional elasticity theory should be used which can provide results with high accuracy. Based on the two-dimensional elasticity theory, Chen et al. [7] presented a mixed method to analyze the bending and free vibration of uniform beams resting on Pasternak elastic foundation. Xu and Wu [8] provided a two-dimensional analytical solution for simply supported composite beams with interlayer slips. Sankar [9] obtained an elasticity solution for a functionally graded beam subjected to transverse loads. Lu et al. [10] presented semi-analytical elasticity solutions for thermal deformations of functionally graded beams with various end conditions, using the state space method based on differential quadrature. Xu and Zhou [11,12] obtained the elasticity solution of simply-supported functionally graded beams and multi-span beams with variable thickness, respectively. Moreover, Xu et al. [13] presented the elasticity solution of clamped-simply supported beams with variable thickness. Recently, Xu and Zhou [14] presented two-dimensional analysis of simply supported piezoelectric beams with variable thickness.

Thermoelasticity is often used to refer various phenomena associated with the interaction between deformation and heat exchange occurring in a body. A monograph about coupled thermoelasticity was published by Biot [15]. Li and Zhou [16] carried out the geometrically nonlinear analysis on Timoshenko beams under thermo-mechanical loads. Tanigawa et al. [17] made the transient thermal stress analysis of a laminated composite beam. Sun et al. [18] studied the thermo-mechanical deformation of an elastic beam with embedded shape memory alloy wires. Sayman [19] carried out an elastic–plastic thermal stress analysis on steel fiber-reinforced aluminum metal–matrix composite beams. Kapuria et al. [20] presented a new efficient higher order zigzag theory for thermal stress analysis of laminated beams subjected to thermal loads. Ching and Yen [21] studied the transient thermoelastic deformations of 2-D functionally graded beams with non-uniformly convective heat supply. Lu et al. [22] presented the two-dimensional thermoelasticity solution for functionally graded thick beams. Vidal and Polit [23] derived a three-noded thermo-mechanical beam finite element to analyze the laminated beams. Alibeigloo [24] presented an analytical solution for functionally graded material beams integrated with piezoelectric actuator and sensor under an applied electric field and thermo-mechanical load. Ma and Lee [25] obtained an exact closed-form solution for the nonlinear static responses of beams made of functionally graded materials subjected to a uniform in-plane thermal load. Recently, Xu et al. [26] presented the three-dimensional thermoelastic analysis of rectangular plates with variable thickness subjected to thermo-mechanical loads.

In this paper, the displacement and stress distributions of simply supported beams with variable thickness subjected to thermo-mechanical loads are studied based on the two-dimensional thermoelastic theory. The temperature field is expressed as a harmonic series whereas the nonlinear temperature profile along the thickness is determined by solving the heat conduction equation. Two typical boundary conditions on the beam ends are considered, which corresponds to two different series expansions. On the basis of two-dimensional thermoelasticity theory, the solutions of displacements and stresses of the beam, which satisfy the governing differential equations and the boundary conditions at two ends of the beam, are analytically derived. The unknown coefficients in the solutions are approximately determined by using the Fourier sinusoidal series expansions to the upper surface and lower surface conditions of the beam. Solutions with high accuracy are presented.

2. Basic formulations

Consider a continuously varying thickness beam with the length L and the thickness H at the left end, as shown in Fig. 1. The beam is simply supported at the two ends $x = 0$ and $x = L$ (called as S–S beam). The upper surface of the beam is horizontal and subjected to a steady state temperature load $t(x)$ and a mechanical load $q(x)$. The profile of its lower surface is described by the continuous functions $f(x)$.

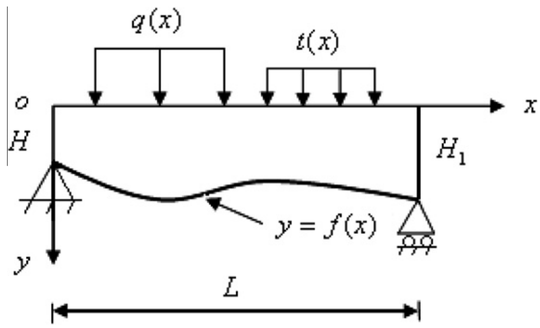

Fig. 1. Beam with continuously varying thickness.

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