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Discretization of nonlinear input-driven dynamical systems using the Adomian Decomposition Method

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ABSTRACT

A numerical decomposition method proposed by Adomian provides solutions to nonlinear, or stochastic, continuous time systems without the usual restrictive restraints. It is applicable to differential, delay differential, integro-differential, and partial differential equations without the need for linearization or other restrictions. It also works through both uncoupled boundary conditions as well as delay systems. In the following paper, a new time discretization method for the development of a sample-data representation of a nonlinear continuous-time input-driven dynamical system is proposed. The proposed method is based on both the zero-order hold (ZOH) assumption as well as the Adomian Decomposition Method which exhibit unique algorithmic and computational advantages. To take advantage of this method, the following steps must be followed. First, the method is applied to a linear input-driven dynamical system to explicitly derive an exact sample-data representation, producing proper results. Second, the method is then applied to a nonlinear input-driven dynamical system, which thereby derives exact and approximate sample-data representations, the latter being most suited for practical applications. To evaluate the performance, the proposed discretization procedure was tested using simulations in a case study which involved an illustrative two-degree-of-freedom mechanical system that exhibited nonlinear behavior considering various control and input variable profiles. In conclusion, the suggested algorithm, in comparison to the results of a Taylor-Lie series expansion method, demonstrated increased performance and efficiency.

1. Introduction

Due to the exponential development and widespread use of digital technology in the global market, an extensive implementation of advanced computer-based control and condition monitoring systems has infiltrated practically all major industrial sectors. In particular, recent advances in digital microprocessor technology have brought considerable merit to digital computer-controlling and monitoring systems with relatively low operational costs, flexible implementation, and simple yet functional interactive communications through several control loops. Consequently, motivation to push the further development of direct digital processes, system controls, and supervision methods remains a crucial point [1–3].

In current practice, advanced control strategies are employed for the dynamic behaviors of complex system processing inducing the desirable sets of dynamic characteristics usually implemented using microcontrollers or digital signal processors.

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On the other hand, the associated control algorithms are developed in a discrete-time domain only. Below are two alternative approaches that could be utilized in designing more productive digital computer-control systems:

(1) Develop a continuous-time control law on the basis of original continuous-time input-driven systems, followed by the discretization of the closed-loop dynamics and the implementation of the above control law digitally through rapid sampling. This method is based on the methodological principles of the so called discrete-equivalent design [4]. Notice that the emulation of the continuous compensators with digital equivalents is a popular design method, used quite extensively by control engineers [5–7].
(2) Obtain a sample-data representation of the original continuous-time input driven system (i.e. an accurate representation of its dynamics in the discrete-time domain directly through the employment of identification methods or a reliable time-discretization method) and then directly synthesize a discrete-time controller to enforce the desirable dynamic behavior to the controlled process/system [1,4,8,9]. This approach offers and attractive feature of dealing directly with the sampling issue at hand. Indeed, the effect of sampling on the system-theoretic properties of a continuous-time system is crucial due to its strong influences in the ability to meet the primary control design objectives [8,10–12].

It should be noted that, in both design approaches, the time-discretization of either controller's or the original systems' dynamic equations is required. However, for reasons stated previously, the second approach will be used in this study.

When using the linear systems theory, the problem of time-discretization of a continuous-time linear input-driven dynamical system under the zero-order hold (ZOH) assumption can be fully solved. Thereafter, an exact sample-data representation of the original system may be obtained in a straightforward manner [13]. In this, two distinguishable and equivalent approaches may be considered:

(1) Capitalizing on the convolutional integral properties under a constant input value over the sampling interval. This approach is based on (1) an integral-operator representation of the input/output behavior of the system (realized by the convolution integral) and (2) appears to be quite effective for theoretical investigations on important system-theoretic properties of the resulting sampled-data representation [14].
(2) Using this approach, the original problem is mathematically reformulated as a linear autonomous equation under the zero-hold-order assumption, for which the standard solution techniques from linear, autonomous systems theory are employed [8].

Most processes and systems encountered in practical uses are inherently nonlinear. As it is well known, (bio) chemical processes, computer networks, electrical and mechanical systems, and bio-systems are all governed by incredibly complex dynamics. Any attempt to locally linearize these processes may cause poor identification and ultimately alternate behavior [15]. Consequently, nonlinear controller synthesis' and monitoring system design problems have been extensively explored and researched over a range of years [16–20]. Furthermore, in light of the above considerations, the viability and performance of digitally computer-based controls and monitoring supervision strategies critically depend on the selection and properties of the underlying time-discretization methods that are applied to the original continuous-time input-driven dynamical systems and processes. In the field of nonlinear systems analysis, the time-discretization problem is both difficult and challenging. Traditionally, approximate sample-data representations of the original continuous-time nonlinear dynamical system is obtained using some popular numerical techniques such as Euler and Runge-Kutta [21], which are suitable for the integration of nonlinear Ordinary Differential Equations (ODE's). However, all of these approaches require a "small" time step in order to accurately function. This may not be the case in many process control and monitoring applications where large sampling periods are inevitably introduced due to physical and technical limitations. A thorough but non-exhaustive sample of differing approaches with practical limitations is reported in [11,22]. Similarly, solid theoretical results based on the direct use of discrete-time approximations in the control of sample-data nonlinear systems can be located in [12,23].

Notice that the sampling period can be selected only after the analogue control system is designed, to ensure that the continuous-time closed-loop bandwidth is known. Performance of this method is significantly affected by both the method of discretization and the chosen sampling intervals. For example, standard methods such as the bilinear transformations often require high sampling rates to achieve adequate performance and retain closed-loop stability. In more specific cases, in contrast, the sampling rate is constrained computational speeds of the microprocessor for digital control or by the measurement scheme, for which low values are required [24].

Given that, the availability of a time-discretization scheme subject to low sampling rates and nonlinear control, accurate discrete-time dynamic models for input-driven processes and systems would be beneficial in designing effective digital controls and monitoring systems. Therefore, using the same concept, accurate dynamic models using large sampling rates under nonlinear control can establish effective designs for future digital control and monitoring. To summarize, these actions fulfill the purpose of a new time-discretization method for the nonlinear input-driven dynamical systems as outlined in the beginning of this paper.

A numerical decomposition method proposed by Adomian provides solutions to nonlinear, or stochastic, continuous time systems without the usual restrictive restraints. It is applicable to differential, delay differential, integro-differential, and