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Stationary analysis of a discrete-time *GI*/*D-MSP*/1 queue with multiple vacations

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A B S T R A C T

This paper analyzes the steady-state behavior of a discrete-time single-server queueing system with correlated service times and server vacations. The vacation times of the server are independent and geometrically distributed, and their durations are integral multiples of slot duration. The customers are served one at a time under discrete-time Markovian service process. The new service process starts with the initial phase distribution independent of the path followed by the previous service process when the server returns from a vacation and finds at least one waiting customer. The matrix-geometric method is used to obtain the probability distribution of system-length at prearrival epoch. With the help of Markov renewal theory approach, we also derive the system-length distribution at an arbitrary epoch. The analysis of actual-waiting-time distribution in the queue measured in slots has also been carried out. In addition, computational experiences with a variety of numerical results are discussed to display the effect of the system parameters on the performance measures.

1. Introduction

Queueing models with non-renewal service processes are often used to model complex computer and communication systems. Several connections (data, voice, video, etc.) generate traffic streams with very different characteristics (required bandwidth, burstiness, correlation, etc.). Traditional teletraffic analysis based on exponential/geometric service time distribution is not powerful enough to capture this correlative and bursty feature of traffic streams in high-speed packet/cell based networks. These correlated and bursty non-renewal service processes in queueing systems have been shown empirically and theoretically to have a significant impact on queueing behavior. The discrete-time Markovian service process (*D*-MSP) is a versatile service process and can capture the correlation among successive service times. Note that the *D*-MSP is independent of the arrival process and it generates real service completions only when the server is busy. This service process forms a rich class of point processes that include several known processes such as Markov-modulated Bernoulli process (MMBP), discrete-time phase type renewal process, and superposition of such processes.

In the past decades, many researchers have analyzed several queueing models with various types of service processes and are available in the literature. Alfa et al. [1] discussed the asymptotic behavior of the *GI*/*MSP*/1 queue using perturbation theory. The analysis of finite- and infinite-buffer *G*/*MSP*/1 queue has been carried out by Bocharov et al. [2], wherein they derived stationary characteristics of system performance by considering that the service phase does not change in an idle period. Gupta and Banik [3] have analyzed *GI*/*MSP*/1 queue with finite as well as infinite buffer using a combination of

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the matrix-geometric method, the imbedded Markov chain and the supplementary variable techniques. Samanta et al. [4] carried out discrete-time *GI*/*D-MSP*/1 queue with finite and infinite buffers, where the phase of the Markovian service process is not frozen, that is, service phase keeps changing when the server is idle. Wang et al. [5] analyzed the packet loss pattern of the finite-buffer *D-BMAP*/*D-MSP*/1 queueing system using matrix-geometric approach. In these connections, see also Alfa [6] and Horváth et al. [7].

Queueing models with server vacations are characterized by the fact that the idle time of the server may be utilized for some other jobs. During the past decades, queueing systems with different types of vacation policies have been extensively studied by numerous researchers and applied to flexible manufacturing environments, production, computers, communication networks, telecommunication systems, traffic concentrators and other related areas. Some discrete-time vacation queueing models' applications has been reported by Alfa [8]. A variety of vacation policies as well as a vast amount of related references are available in Takagi [9], and Tian and Zhang [10]. Samanta [11] carried out analysis of discrete-time *GI*/*Geo*/1 queue with single vacation. Ozawa [12] first investigated different vacation policies queueing model under Markovian service process, and obtained a matrix-type factorization of the vector generating function for the stationary queue length in *MAP*/*MSP*/1 queues.

In this paper, we analyze an infinite-buffer discrete-time *GI*/*D-MSP*/1 multiple vacations and exhaustive service queueing system, where the input follows a discrete-time renewal process and the departures form a discrete-time Markovian service process. The new service process starts with the initial phase distribution independent of the path followed by the previous service process when the server returns from a vacation and finds at least one waiting customer. The matrix-geometric method is applied to obtain prearrival epoch probability and the Markov renewal theory approach is used to develop a relation between prearrival and arbitrary epoch probabilities. The analysis of actual-waiting-time distribution in the queue measured in slots has also been investigated. In addition, computational experiences with a variety of numerical results are discussed to display the effect of the system parameters on the performance measures. The model presented in this paper may be useful for the performance evaluation of an energy-aware medium access control (MAC)/physical (PHY) layer protocol in view of bursty traffic service patterns (modeled as *D*-MSP). The MAC/PHY layer in a node is modeled as a server and a vacation queueing model is utilized to represent the sleep and wakeup mechanism of the server. Energy efficiency is a major concern in traditional wireless networks due to the limited battery power of the nodes. In one direction to save energy in such a network is to bring into play an efficient sleep and wakeup mechanism to turn off the radio transceiver irregularly in order that the desired trade-off between the node energy savings and the network performance can be achieved.

This paper is organized as follows. In Section 2, we give the description of the model and introduce the notations to describe the model parameters. The steady-state system-length distributions at various epochs and waiting-time distribution of an arriving customer are analyzed in Section 3. A variety of numerical results are presented in Section 4. Section 5 concludes the paper.

2. Model description and notations

We consider a single-server infinite-buffer queueing system wherein customers are served according to a discrete-time Markovian service process (*D*-MSP). Formally, *D*-MSP is characterized by the services which are governed by an underlying m-state Markov chain having probability $(L_0)_{ij}$, $1 \leqslant i, j \leqslant m$, of a transition from state i to j without service completion and probability $(L_1)_{ij}$, $1 \leqslant i, j \leqslant m$, of a transition from state i to j with service completion. Let $(L_0)_{ij}$ and $(L_1)_{ij}$ be the (i,j)-th entry of the $m \times m$ non-negative matrices $\mathbf{L}_0$ and $\mathbf{L}_1$, respectively, with $\mathbf{L}_1$ having at least one positive entry such that $(\mathbf{L}_0 + \mathbf{L}_1)\mathbf{e} = \mathbf{e}$, where $\mathbf{e}$ is a column vector of ones with an appropriate dimension. The sum $(\mathbf{L}_0 + \mathbf{L}_1)$ is a stochastic matrix corresponding to the transition probability matrix of an irreducible Markov chain underlying *D*-MSP. We call the actual state of this chain the "phase" of the service process. Let $\overline{\boldsymbol{\pi}} = [\overline{\pi}_1, \overline{\pi}_2, \cdots, \overline{\pi}_m]$ be the stationary probability vector of the Markov process with the stochastic matrix $(\mathbf{L_0} + \mathbf{L_1})$, where $\overline{\pi}_j$ denotes the steady-state probability of service process being in phase j. The stationary probability row-vector $\overline{\boldsymbol{\pi}}$ can be calculated from $\overline{\boldsymbol{\pi}}(\mathbf{L_0} + \mathbf{L_1}) = \overline{\boldsymbol{\pi}}$ with $\overline{\boldsymbol{\pi}}\mathrm{e} = 1$. Let $Y(t)$ denote the number of customers served in the first t time slots when the server is busy and $I(t)$ the state of the underlying Markov chain (called service phase) after the same amount of time. Then $\{Y(t), I(t)\}_{t \geqslant 0}$ is a two-dimensional discrete-time Markov process with state space $\{(n,i) : n \geqslant 0, 1 \leqslant i \leqslant m\}$ and the state transition matrix

$$\mathbf{Q}^{\mathrm{D-MSP}} = \begin{pmatrix} \mathbf{L}_0 & \mathbf{L}_1 & \mathbf{0} & \mathbf{0} & \cdots \\ \mathbf{0} & \mathbf{L}_0 & \mathbf{L}_1 & \mathbf{0} & \cdots \\ \mathbf{0} & \mathbf{0} & \mathbf{L}_0 & \mathbf{L}_1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

Let $\{\mathbf{P}(n,t) : n \geqslant 0, t \geqslant 1\}$ denote the matrix of order $m \times m$ whose (i,j)-th element $(P_{i,j}(n,t))$ represents the probability that n customers are served in $(0,t]$ with the service process being in phase j at time t, provided initially there were at least $(n+1)$ customers (including the one in service) in the system and the service process was in phase i. Then, the probabilities

$$P_{ij}(n,t) = Pr\{Y(t) = n, \quad I(t) = j | Y(0) = 0, I(0) = i\}, \quad 1 \leqslant i,\ j \leqslant m,$$

lead to the following equations (in matrix notation)