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Evaluation of fully fuzzy matrix equations by fuzzy neural network

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ABSTRACT

In this paper, a new hybrid method based on fuzzy neural network for approximate solution of fully fuzzy matrix equations of the form $AX = D$, where A and D are two fuzzy number matrices and the unknown matrix X is a fuzzy number matrix, is presented. Then, we propose some definitions which are fuzzy zero number, fuzzy one number and fuzzy identity matrix. Based on these definitions, direct computation of fuzzy inverse matrix is done using fuzzy matrix equations and fuzzy neural network. It is noted that the uniqueness of the calculated fuzzy inverse matrix is not guaranteed. Here a neural network is considered as a part of a large field called neural computing or soft computing. Moreover, in order to find the approximate solution of fuzzy matrix equations that supposedly has a unique fuzzy solution, a simple algorithm from the cost function of the fuzzy neural network is proposed. To illustrate the easy application of the proposed method, numerical examples are given and the obtained results are discussed.

1. Introduction

The concept of fuzzy numbers and fuzzy arithmetic operations were first introduced by Zadeh [1], Dubois and Prade [2]. We refer the reader to [3] for more information on fuzzy numbers and fuzzy arithmetic. Fuzzy systems are used to study a variety of problems ranging from fuzzy topological spaces [4] to control chaotic systems [5,6], fuzzy metric spaces [7], fuzzy linear systems [8–15] and particle physics [16–19].

One of the major applications of fuzzy number arithmetic is to treat fully fuzzy linear systems. Several problems in various areas such as economics, engineering and physics lead to the solution of a linear system of equations. In many applications, the parameters of the system (or at least some of them) should be represented by fuzzy rather than crisp numbers. Thus, it is very important to develop numerical procedures that can appropriately treat fuzzy linear systems and solve them.

Friedman et al. [20] introduced a general model for solving a fuzzy $n \times n$ linear system whose coefficient matrix is crisp and the right-hand side column is an arbitrary fuzzy number vector. They used the parametric form of fuzzy numbers and replaced the original fuzzy $n \times n$ linear system by a crisp $2n \times 2n$ linear system and studied the duality in fuzzy linear systems $Ax = Bx + y$ where A and B are two real $n \times n$ matrices and the unknown x and the known y are two vectors whose components are n fuzzy numbers [21]. In [8,9,14,22] the authors presented conjugate gradient and LU decomposition method for solving general fuzzy linear systems or symmetric fuzzy linear systems. Also, Wang et al. [23] presented an iterative algorithm for solving dual linear systems of the form $x = Ax + u$, where A is a real $n \times n$ matrix, the unknown x and the constant u are all vectors whose components are fuzzy numbers. Abbasbandy et al. [24] investigated the existence of a minimal solution of a general dual fuzzy linear system of the form $Ax + f = Bx + c$, where A and B are two real $m \times n$ matrices and the unknown x and the known f and c are vectors whose components are fuzzy numbers. Recently, Muzziloi et al. [25] considered fully

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fuzzy linear systems of the form $A_1x + b_1 = A_2x + b_2$ with A_1 and A_2 two square matrices of fuzzy entries and b_1 and b_2 fuzzy number vectors. Dehghan et al. [26] considered fully fuzzy linear systems of the form $A \otimes x = b$ where A is a positive fuzzy matrix, x is an unknown and b is a known positive fuzzy vector respectively.

Ishibuchi et al. [27] proposed a learning algorithm of fuzzy neural networks with triangular fuzzy weights and Hayashi et al. [28] fuzzified the delta rule. Buckley and Eslami [29] considered neural net solutions to fuzzy problems. Fuzzy neural network have been extensively studied [30–32] and recently, successfully used for solving fuzzy polynomial equation and systems of fuzzy polynomials [33–35], approximate fuzzy coefficients of fuzzy regression models [36–38], approximate solution of fuzzy linear systems and fully fuzzy linear systems [39,40], fuzzy differential equations and differential equations [41,42]. One of the major applications of fuzzy neural networks is treating control and synchronization of chaos [43,44].

In this paper, we first propose an architecture of fuzzy neural network (FNN) with fuzzy weights for fuzzy input vectors and fuzzy targets to find approximate solution to fully fuzzy matrix equations (FFMEs) like $AX = D$. The input–output relation of each unit is defined by the extension principle of Zadeh [1]. Output from the fuzzy neural network, which is also a fuzzy number, is numerically calculated by interval arithmetic [48] for fuzzy weights and level sets (i.e., h-cuts) of fuzzy inputs. Next, we define a cost function for the level sets of fuzzy outputs and fuzzy targets. Then, a crisp learning algorithm is derived from the cost function to find the fuzzy solutions of the FFME. Section 5, includes some new definition pertinent to calculating the inverse of a fuzzy matrix, that the calculate inverse of fuzzy matrix is the one of the application of FFMEs. The proposed algorithm is illustrated by some examples in the last section.

2. Preliminaries

In this section the basic notations used in fuzzy calculus are introduced. We start by defining the fuzzy number.

Definition 1 [45]. A fuzzy number is a fuzzy set $u : \mathbb{R}^1 \to I = [0,1]$ such that

(i) u is upper semi-continuous.
(ii) $u(x) = 0$ outside some interval $[a,d]$.
(iii) There are real numbers b and c, $a \leqslant b \leqslant c \leqslant d$, for which
  1. $u(x)$ is monotonically increasing on $[a,b]$,
  2. $u(x)$ is monotonically decreasing on $[c,d]$,
  3. $u(x) = 1,\ b \leqslant x \leqslant c$.

The set of all the fuzzy numbers (as given in Definition 1) is denoted by E^1.

An alternative definition which yields the same E^1 is given by Kaleva [46,47].

Definition 2. A fuzzy number u is a pair $(\underline{u}, \overline{u})$ of functions $\underline{u}(r)$ and $\overline{u}(r)$, $0 \leqslant r \leqslant 1$, which satisfy the following requirements:

(i) $\underline{u}(r)$ is a bounded monotonically increasing, left continuous function on $(0,1]$ and right continuous at 0.
(ii) $\overline{u}(r)$ is a bounded monotonically decreasing, left continuous function on $(0,1]$ and right continuous at 0.
(iii) $\underline{u}(r) \leqslant \overline{u}(r),\ 0 \leqslant r \leqslant 1$.

A crisp number r is simply represented by $\underline{u}(\alpha) = \overline{u}(\alpha) = r,\ 0 \leqslant \alpha \leqslant 1$. The set of all the fuzzy numbers is denoted by E^1.

A popular fuzzy number is the triangular fuzzy number $u = (u_m, u_l, u_r)$ where u_m denotes the modal value and the real values $u_l > 0$ and $u_r > 0$ represent the left and right fuzziness, respectively. The membership function of a triangular fuzzy number is defined by:

$$\mu_u(x) = \begin{cases} \frac{x - u_m}{u_l} + 1, & u_m - u_l \leqslant x \leqslant u_m, \\ \frac{u_m - x}{u_r} + 1, & u_m \leqslant x \leqslant u_m + u_r, \\ 0, & \textit{otherwise}. \end{cases}$$

Its parametric form is

$$\underline{u}(\alpha) = u_m + u_l(\alpha - 1), \quad \overline{u}(\alpha) = u_m + u_r(1 - \alpha).$$

Triangular fuzzy numbers are fuzzy numbers in LR representation where the reference functions L and R are linear. The set of all triangular fuzzy numbers on $\mathbb{R}$ is called $\hat{FZ}$.

2.1. Operations on fuzzy numbers

We briefly mention fuzzy number operations defined by the extension principle [1]. Since input vector of feedforward neural network is fuzzified in this paper, the operations we use in our fuzzy neural network are fuzzified by means of the extension principle as follows: