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Perturbation method for solving the nonlinear eigenvalue problem arising from fatigue crack growth problem in a damaged medium

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ABSTRACT

An analytical solution of the nonlinear eigenvalue problem arising from the fatigue crack growth problem in a damaged medium in coupled formulation is obtained. The perturbation technique for solving the nonlinear eigenvalue problem is used. The method allows to find the analytical formula expressing the eigenvalue as the function of parameters of the damage evolution law. It is shown that the eigenvalues of the nonlinear eigenvalue problem are fully determined by the exponents of the damage evolution law. In the paper the third-order (four-term) asymptotic expansions of the angular functions determining the stress and continuity fields in the neighborhood of the crack tip are given. The asymptotic expansions of the angular functions permit to find the closed-form solution for the problem considered.

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1. Introduction

Nowadays in fracture mechanics the eigenfunction expansion method for the description of stress state near a crack tip is in most common use. The approach goes back to the Williams's papers [1,2], where considering the asymptotic stress field at the vicinity of re-entrant corners and cracks in isotropic, linear, elastic plane media Williams proposed the eigenfunction expansion method and the Airy stress function $\Phi(r,\theta)$ as $r \to 0$ is decomposed into a radial and an angular part

$$\Phi(r,\theta) = \sum_j r^{\gamma_j} f_j(\theta,\gamma_j), \tag{1.1}$$

where γ_j and $f_j(\theta,\gamma_j)$ are determined according to an eigenproblem and are the eigenvalues and eigenfunctions respectively.

Since then the asymptotic analysis of the stress field near a crack tip is an integral part of fracture mechanics analysis [3,4] for both linear elastic media and nonlinear media (for cracks in power-law hardening materials, for cracked structures under creep conditions). Only in the very recent past the eigenfunction expansion method for the description of stress–strain state near a crack tip is used in a number of papers [5,6,8–12]. In [5] a new boundary element approach is proposed to determine the singular stress field in plane V-notch structures. The method is based on an asymptotic expansion of the stresses in a small region around a notch tip and application of the conventional boundary element approach in the remaining region of the structure. The evaluation of stress singularities at a notch tip is transformed into an eigenvalue problem of ordinary differential equations that is solved by the interpolating matrix method in order to obtain singularity orders (degrees) and

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Nomenclature

σ_{ij}	stress components
ε_{ij}	strain components
ψ	continuity parameter
D	damage parameter
s	order of the stress singularity
x, y	Cartesian coordinates
r	radius in polar coordinates at crack tip
θ	angle in polar coordinates at crack tip
a	current crack length
γ_j	eigenvalues of the Williams expansion
$\sigma_{ij}^{(k)}$	non-dimensional angular functions in asymptotic expansion for stress field
E	Young's modulus
ν	Poisson's ratio
g	angular function for continuity
λ, μ	powers in stress and continuity fields at crack tip
λ_0, μ_0	powers in stress and continuity fields at crack tip corresponding to the linear "undisturbed" problem
Φ	Airy stress function
e_i, s_i	coefficients of expression for asymptotic strain field
b_i, d_i	coefficients of fourth-order ordinary differential equations with respect to $f(\theta)$
α, β	amplitudes of stress and continuity fields
σ_e	von Mises equivalent stress
$\sigma_e^{(k)}$	angular distributions of the Mises equivalent stress
$\varepsilon_{ij}^{(k)}$	angular distributions of the strain components
N	number of cycles
σ_{th}	positive material constant of the damage evolution law
θ_d	angle separating the process zone and totally damaged zone
f	angular distribution of the Airy stress potential
c, m, n	positive material constants of the damage evolution law
$\bar{E}$	ratio of derivative of continuity to continuity
s	order of stress singularity
ε	small parameter
E_k	coefficients of the four-term expansion of $\bar{E}$
G_k	coefficients of the four-term expansion of $\bar{G}$
n_k	coefficients of the power series of n
m_k	coefficients of the power series of m
λ_k	coefficients of the power series of λ
$f_k(\theta)$	coefficients of the power series of $f(\theta)$
$g_k(\theta)$	coefficients of the power series of $g(\theta)$
b_i^j	coefficients of the power series of b_i
e_i^j	coefficients of the power series of e_i
$\sigma_e^{(k)}(\theta)$	coefficients of the power series of the effective stress $\sigma_e(r, \theta)$
$\varphi_k(\theta)$	adjoint solution
$g^{(k)}(\theta)$	coefficients of the continuity parameter asymptotic expansion
α_1, β_1	amplitude coefficients of the two-term asymptotic expansions

associated eigenfunctions of the V-notch. The combination of the eigenanalysis for the small region and the conventional BE analysis for the remaining part of the structure results in both the singular stress field near the notch tip and the notch stress intensity factors.

A preliminary asymptotic analysis of the stress field near a V-notch tip of bonded dissimilar materials is the basis of the study performed by Niu et al. [6]. According to the linear theory of elasticity, there exists a combination of different orders of stress singularity at a V-notch tip of bonded dissimilar materials. The singularity reflects a strong stress concentration near the sharp V-notches. In the paper, a new way is proposed in order to determine the orders of singularity for two-dimensional V-notch problems. On the basis of an asymptotic stress field in terms of radial coordinates at the V-notch tip, the governing equations of the elastic theory are transformed into an eigenvalue problem of ordinary differential equations with respect to the circumferential coordinate around the notch tip. The main singularity orders of plane V-notches are calculated from solving the eigenvalue problems of the ordinary differential equations. In parallel, the associated eigenvectors of the displacement and stress fields near the V-notches are also determined.

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