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Implementation of near-wall boundary conditions for modeling boundary layers with free-stream turbulence

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The paper is devoted to the extension of the near-wall domain decomposition, earlier developed in some previous works by the authors, to modeling flat-plate boundary layers undergoing laminar-to-turbulent bypass transition. The steady-state wall boundary layers at high-intensity free-stream turbulence are studied on the basis of differential turbulence models with the use of non-overlapping domain decomposition. In the approach the near-wall resolution is replaced by the interface boundary conditions of Robin type. In contrast to the previous studies, the main attention is paid to the laminar–turbulent transitional regime. With the use of modified turbulence models we study an effect of free-stream parameters on the development of dynamic processes in the boundary layer including a transitional regime and fully developed turbulent flow. In addition, for the first time a full scale domain decomposition is realized via iterations between the inner and outer subregions until a convergence. The computational profiles of the velocity and intensity of the turbulence kinetic energy are compared with experimental data. A possible range of location of the near-wall interface boundary is found.

1. Introduction

The problem of mathematical modeling of near-wall turbulent flows is very complicated due to the existence of a thin viscous sublayer by the wall, in which the molecular diffusion dominates above the turbulent. The small sublayer thickness leads to high gradients of the characteristics of a flow. The calculation of heat transfer in this near-wall zone requires an adequate numerical resolution. As a result, in many practical applications a very fine mesh is needed. this leads to a considerable increase of the computational time used. In this case near-wall damping functions are often involved in the governing equations assigned to so called low-Reynolds-number models (LR). The equations of the LR models are applicable in the entire region up to the wall.

Next, we consider the main approaches to resolve this problem. In the first class of methods, the viscous sublayer is not resolved and the boundary conditions are set outside. Such boundary conditions are of Dirichlet type and called the wall functions. They are used to obtain the main characteristics of the flow on the basis of so called high-Reynolds-number models (HR).

The wall functions for the velocity and temperature are often based on the log-law [1,2] and have limited applications. As noted in [1,3], the pressure gradient distribution along a surface and the wall boundary condition for the normal velocity

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should be taken into account in the wall functions. The pressure gradient is taken into account in the analytical and numerical wall functions [4,5]. In these approaches a local solution is obtained in a subgrid area in the nearest to the wall cell. The pressure is assumed to be constant across the sublayer [6]. In contrast to the conventional wall functions the information on the flow in the near-wall subgrid area is used in the numerical approximation of the governing equations. The analytical wall functions [4] are based on the assumption of a piece-wise linear viscosity profile. The numerical wall functions [5] are free from this assumption and, therefore, more universal although more time consuming. The location of the first mesh node, at which the boundary conditions are set, also has a strong influence on the accuracy of the solution. In turn, the scalable wall functions [7] allow one to reduce this effect.

In the second class of approaches the boundary conditions are transferred from the wall to some intermediate surface in a near-wall region. they are represented in the differential form as Robin-type boundary conditions. Such interface boundary conditions can be interpreted as generalized wall-functions [8]. In this case both the HR and LR differential turbulence models can be used. The application of the interface boundary conditions to LR models leads to a domain decomposition [9,10]. It is based on transferring the boundary conditions from the wall to an interface boundary. In contrast to the conventional LR models, the algorithm allows one to avoid computationally expensive calculations related to the near-wall region while retaining a sufficiently high accuracy. As shown in [8,9], the boundary conditions formulated at the interface boundary are uniformly applicable for a wide range of the input parameters. They are also suitable for very different locations of the interface boundary. In comparison with composite models such as [11,12], the developed domain decomposition approach is based on the use of the same model in the inner and outer domains. As a result, in particular, it guarantees a smooth composite solution across the interface. It is worth noting that mesh distributions on both sides of the interface point can be independent from each other. Despite the use of the interface boundary conditions looks attractable and resulted in a reasonably good prediction [8,10], this approach has never been used for modeling flows with laminar–turbulent transition that makes the flow structure very complicated.

If the amplitudes of disturbances are small enough, the transition to turbulence is developed from a number of stages with the rise of Tollmein–Schlichting waves. For a free stream with a higher turbulence intensity large amplitude disturbances diffuse directly into the boundary layer bypassing these stages the with appearance of waves [13].

At a higher turbulence intensity $\mathrm{Tu}_\infty > 1\%$, the transition to turbulence in a boundary layer is accompanied with increasing low frequency strip longitudinal structures [14]. Here, $\mathrm{Tu}_\infty = 10^2\sqrt{2K_\infty/3}/V_\infty$ (%), including free stream parameters: the velocity V_∞ and the kinetic turbulence energy K_∞. If the intensity is still moderate $\mathrm{Tu}_\infty < 5\%$, then these structures have a predominant form of disturbances in the boundary layer. For large disturbances with Tu_∞ near 10%, the turbulent bursts appear that can be described by statistical models [15,16].

According to experimental data and theoretical studies, the turbulent intensity level has a more significant effect on a laminar-turbulent transition than the level of scale L_∞ [17]. In [18–20] the modification of existing one- and two-parametric turbulence models takes into account the effect of intensity and the scale of free stream turbulence as well as the combined action of the turbulence intensity on the heat transfer and near-wall turbulence characteristics. This approach is based on the analysis of experimental data [21,22]. It allows numerical modeling transition and fully developed turbulent regimes in the boundary layer for near-wall flows with high-intensity turbulence.

In the current paper, for the first time the interface boundary conditions, developed in [8,9,23,10,24] for fully developed turbulent boundary layers, are extended to laminar-to-turbulent bypass transition regimes. On the basis of modified turbulence models the effect of high-intensity free-stream turbulence on the development of dynamic processes in laminar-turbulent transition is studied in application to the steady-state boundary layer. In addition, for the first time a full domain decomposition is realized via iterations between the inner and outer subregions.

The paper is organized as follows. In Section 2 the standard formulation of the boundary value problem for the boundary layer equations for compressible gas is given. The governing equations are closed via the use of the K-ε model modified to simulate a laminar-turbulent transition nearby the wall and formulated in Section 3. The interface boundary conditions are considered in Section 4 in which they are modified to be suitable for a laminar–turbulent transition. The used numerical procedure is described in Section 5. The applicability of the interface boundary conditions in a wide range of Reynolds numbers is analyzed in Section 6 via comparison against experimental and computational data.

2. Mean-flow equations

In the coordinate system ξ, ζ associated with the surface of a body, the system of equations for the averaged characteristics of a two-dimensional boundary layer in a compressible gas flow can be represented in the following form:

$$\frac{\partial}{\partial\xi}(\rho u)+\frac{\partial}{\partial\zeta}(\rho v)=0, \tag{2.1}$$

$$u\frac{\partial u}{\partial\xi}+v\frac{\partial u}{\partial\zeta}=-\frac{1}{\rho}\frac{\partial p}{\partial\xi}+\frac{1}{\rho}\frac{\partial}{\partial\zeta}\left[\mu\frac{\partial u}{\partial\zeta}-\rho\langle u'v'\rangle\right],$$

$$u\frac{\partial h}{\partial\xi}+v\frac{\partial h}{\partial\zeta}=\frac{1}{\rho}\frac{\partial}{\partial\zeta}\left[\frac{\lambda}{c_p}\frac{\partial h}{\partial\zeta}-\rho\langle h'v'\rangle\right]+\frac{u}{\rho}\frac{\partial p}{\partial\xi}+\frac{\mu}{\rho}\left(\frac{\partial u}{\partial\zeta}\right)^2-\langle u'v'\rangle\left(\frac{\partial u}{\partial\zeta}\right),$$